12th Feb 18

Electrostatics

Charge | At rest

classmate
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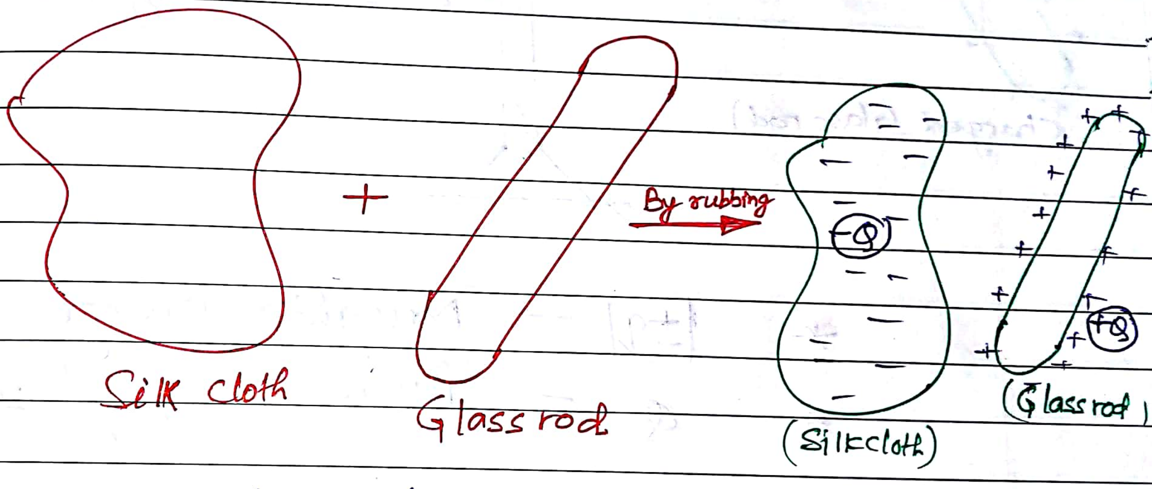
Production of charge:-

(i) → By rubbing / By friction
(ii) → By Induction

(i) By rubbing / By friction :→



→ When we rub a glass rod to silk cloth then glass rod gets ⊕ve charge while silk cloth gets ⊖ve charge.

→ Due to gaining of electrons, ~~charge on~~ mass of silk cloth slightly increases.

→ It is permanent transfer of charge.

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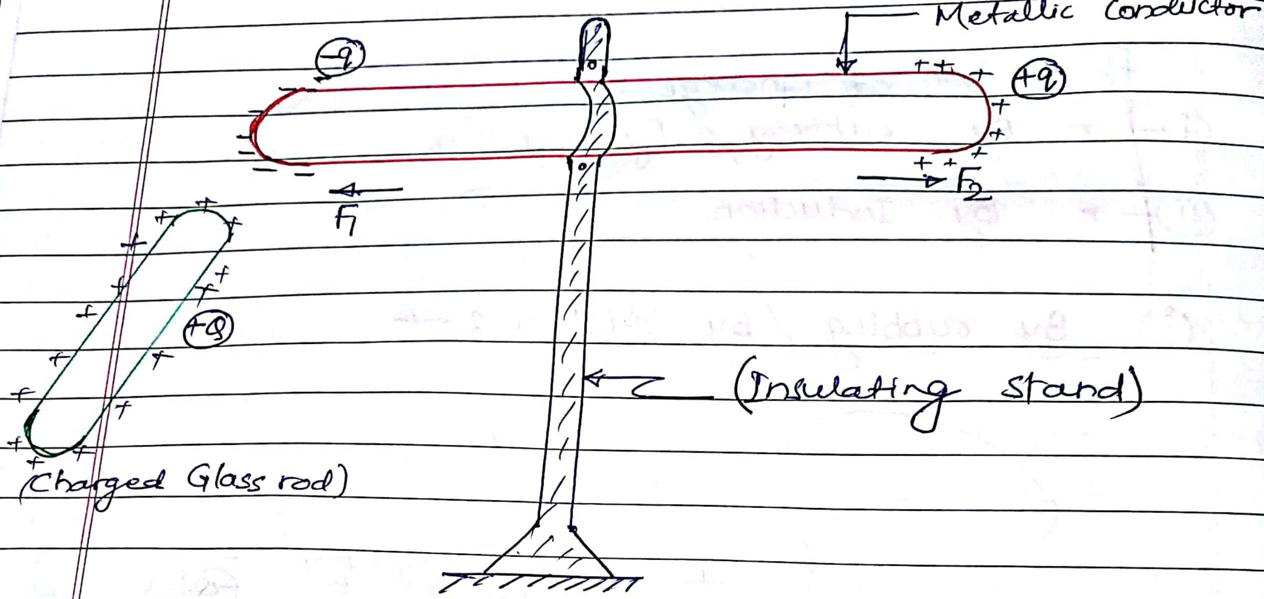
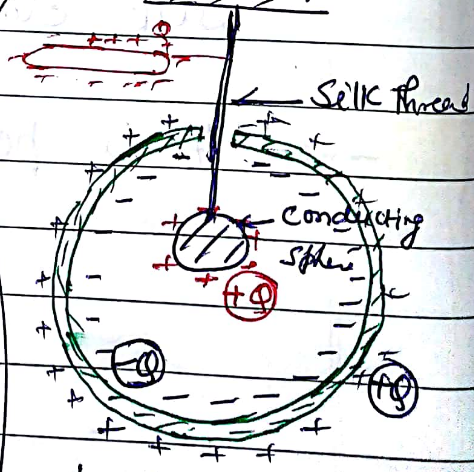
(ii) By induction :-



* $|\pm q|$ = Daughter charge / Induced charge
* Q = Parent charge / Inducing charge
* $|\pm q| \leq Q$
* It is temporary ~~transfer~~ production of charges.
* $F_1 > F_2$ $\left| \left(F \propto \frac{1}{r^2} \right) \right.$

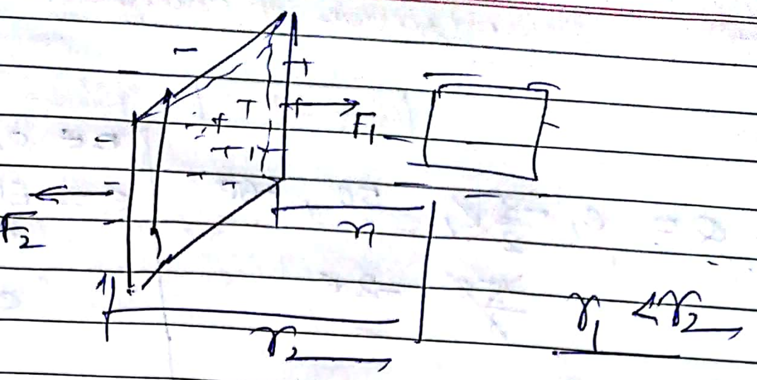
Faraday's result



$|\pm q| = Q$

Note:

(1)



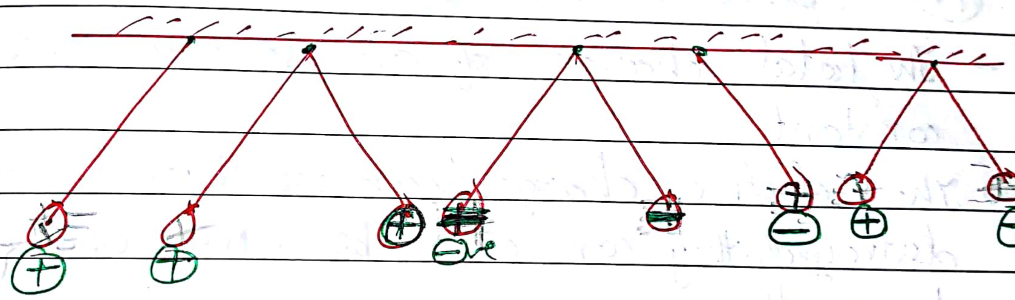
$r_1 < r_2$

Attraction $F_1 > F_2$

(2)

* Properties of charges :→

(i)



(ii) ~~Additivity~~

Additivity:

Total charge of a system is the algebraic sum of all the individual charges located at different points inside the system.

If $Q_1 = 2\mu c$, $Q_2 = -3\mu c$, $Q_3 = +5\mu c$

$\therefore$ Total Charge $Q = Q_1 + Q_2 + Q_3 = [2 + (-3) + 5]\,\mu c$

$Q = 4\mu c$

(iii) Quantization of charges:→

$$\boxed{Q = \pm ne}$$

$n \in 0, 1, 2, 3, ---$

$e =$ Electric charge on an electron

or

$e = 1.6 \times 10^{-19}$ C

$Q = e, \cancel{-\frac{3}{2}e}, 5e, -6e$

$\cancel{\frac{5}{3}e}, -2e$

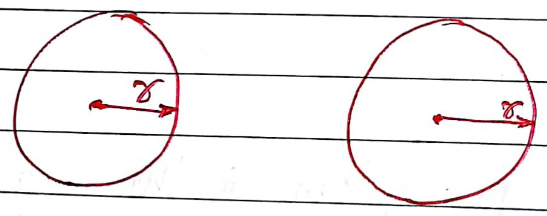
→ Charge on body is always integral multiple of basic charge unit (e).

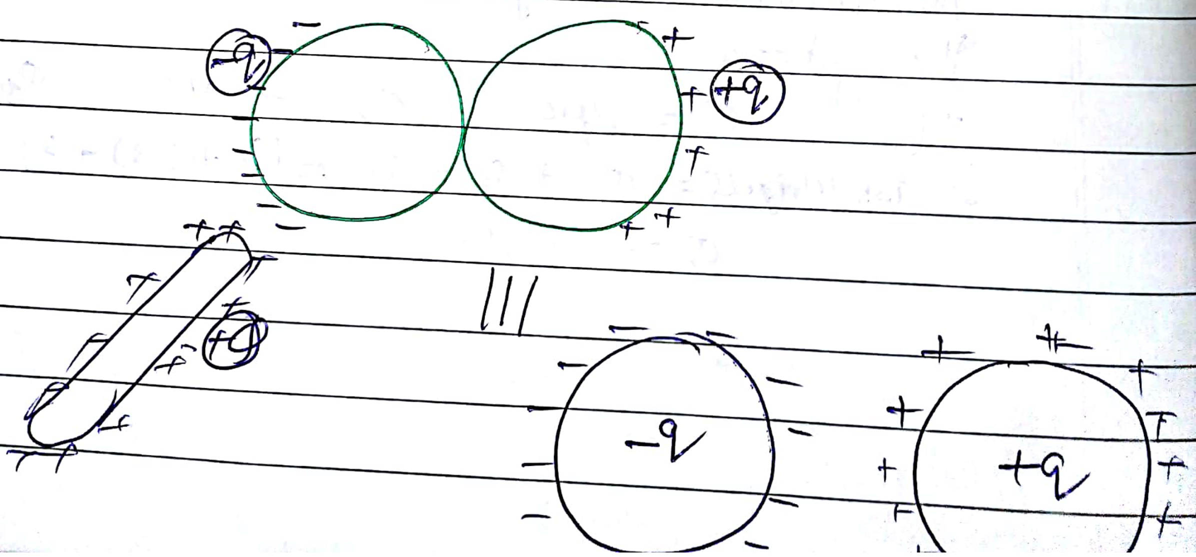
(iv) Conservation of charges:→

→ The total charge of an isolated system remains constant.

→ The electric charges can neither be created nor destroyed, they can only be transferred from one body to another.

Q-

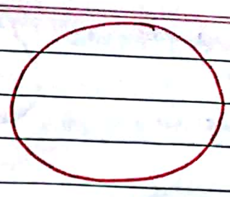


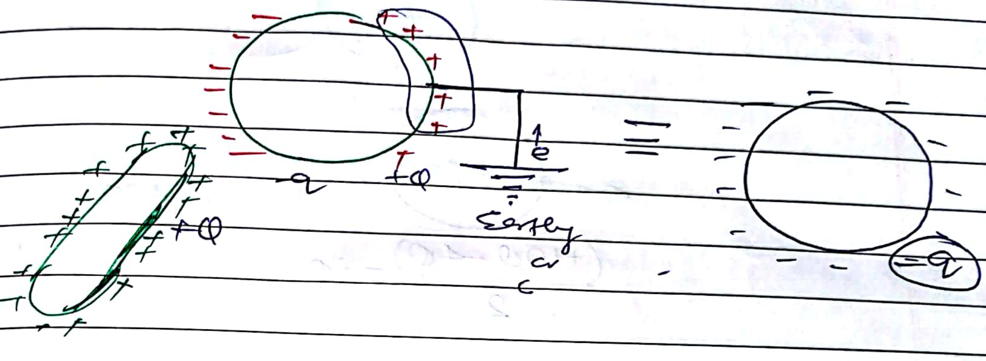


σ = surface charge density

$\sigma \propto \frac{1}{r}$

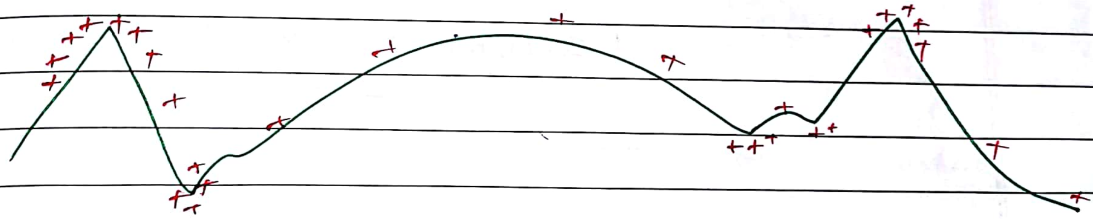
Q.



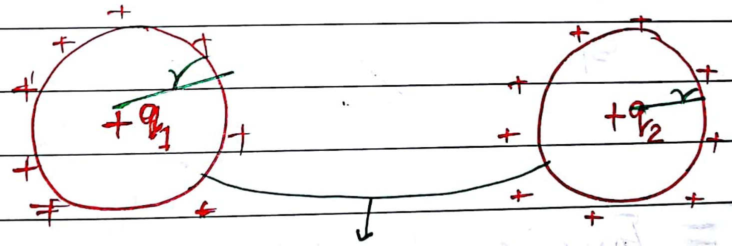


$\sigma \propto \frac{1}{r}$

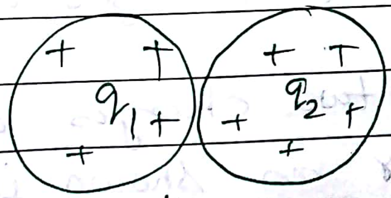
Distribution of charge

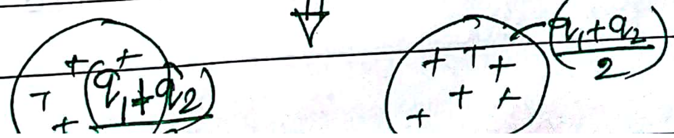


Charge distribution on same shape of body:



Momentary touch





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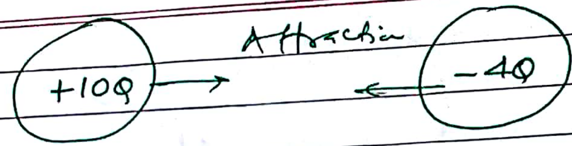
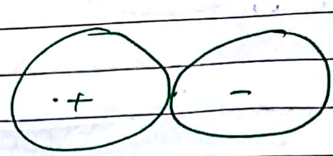
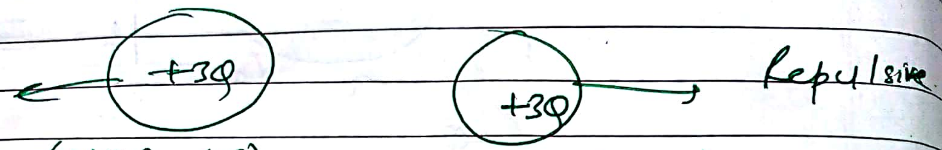






$\frac{(+10Q - 4Q)}{2} = 3Q$ $\qquad$ $\left(\frac{+10Q - 4Q}{2}\right)$

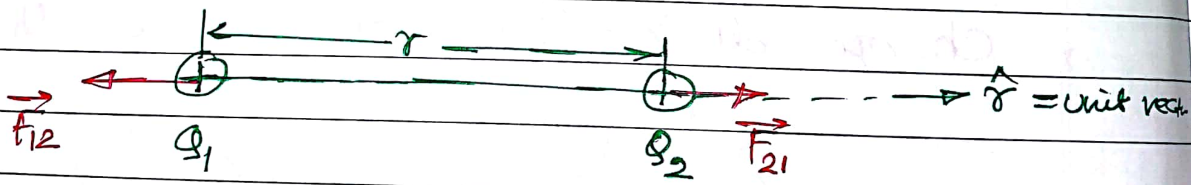
unit of charges:→

1 coulomb $= 3 \times 10^9$ stat coulomb

$= 3 \times 10^9$ esu

Coulomb's Law:→

2M



From Newton's third law:-

$$\boxed{-\vec{F}_{12} = \vec{F}_{21}}$$

$\vec{F}_{12}$ = Force on charge 'Q_1' due to charge 'Q_2'.

$\vec{F}_{21}$ = Force on charge 'Q_2' due to charge Q_1.

Consider two charges 'Q_1' and 'Q_2' placed at distance 'r' as shown in figure.

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Now,
$$|\vec{F_{12}}| = |\vec{F_{21}}| = F \text{ (say)}$$

After coulombian experiment.

$$F \propto (Q_1 \cdot Q_2)$$

$$F \propto \frac{1}{r^2}$$

$$\rightarrow \boxed{F \propto \left(\frac{Q_1 Q_2}{r^2}\right)} \leftarrow \text{coulomb's law.}$$

$$F = k \frac{Q_1 Q_2}{r^2}$$

Value of proportionality constant $= k = \frac{1}{4\pi\epsilon_0}$

$\epsilon_0 =$ Permittivity constant in vacuum.

$$\epsilon_0 = 8.854 \times 10^{-12} \left(C^2/N \cdot m^2\right).$$

$$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \left(\frac{Nm^2}{C^2}\right)$$

$$\therefore \boxed{F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}} \leftarrow \text{For Numerical.}$$

In vector form:

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} \cdot \hat{r}$$

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} \cdot \left(\frac{\vec{r}}{r}\right)$$

$$\therefore \boxed{\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^3} \cdot \vec{r}}$$

Imp ✱) Permittivity and Relative permittivity :→

Permittivity is a property of medium which determines the electric force b/w the two charges situated in that medium.

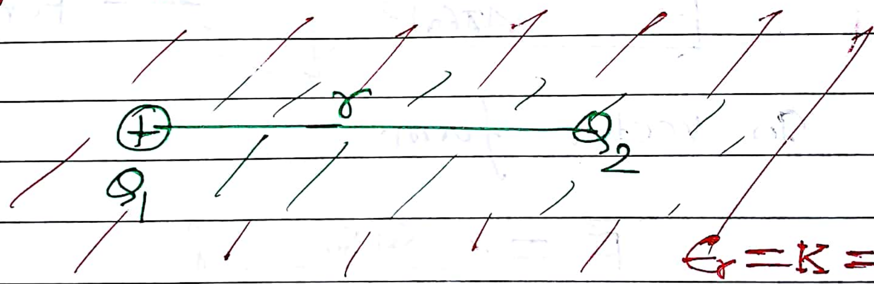
✱ Relative permittivity or dielectric constant :→

$$\boxed{\epsilon_r = \left(\frac{\epsilon_m}{\epsilon_0}\right)} = \left(\frac{\text{Permittivity of medium}}{\text{Permittivity of vacuum}}\right)$$

$$\boxed{\epsilon_r = \left(\frac{F_{vac}}{F_{med}}\right) = \left(\frac{\text{Force betn charge in vacuum}}{\text{Force betn charge in medium}}\right)}$$

$$\boxed{\epsilon_r > 1} \quad \text{or} \quad \boxed{K > 1}$$

✱ Effect of dielectric constant / Effect on force in different ~~field~~ medium :→



$\epsilon_r = K =$ Dielectric Constant or Relative permittivity

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_m \cdot r^2}$$

$$\boxed{\epsilon_m = \epsilon_0 \cdot \epsilon_r}$$

$$\boxed{\epsilon_m = K \cdot \epsilon_0}$$

$$\boxed{F' = \frac{Q_1 Q_2}{4\pi\epsilon_0\epsilon_r r^2} = \frac{Q_1 Q_2}{4\pi\epsilon_0 K r^2}}$$

$$\boxed{F' = \left(\frac{F}{K}\right) = \frac{F}{\epsilon_r}} \quad \because \epsilon_r > 1$$

$$\boxed{F' < F}$$

Hence, ~~dielectric constant~~ new force is less than origional force.

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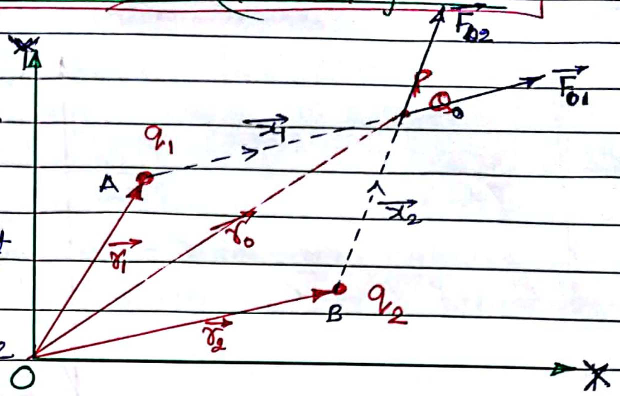
Superposition principle :-

~~Coulomb force in vector form :-~~

Statement:- Force on any charge due to a no. of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges.



Consider three charges q_1, q_2 and 'Q_0' whose position vector $\vec{r_1}$, $\vec{r_2}$ & $\vec{r_0}$ as shown in fig.

In $\triangle OAP$

$$\vec{r_1} + \vec{x_1} = \vec{r_0}$$

$$\Rightarrow \vec{x_1} = (\vec{r_0} - \vec{r_1})$$

$$\therefore \boxed{\vec{x_1} = \vec{r_{01}}}$$

Similarly $\boxed{\vec{x_2} = \vec{r_{02}}}$

Now, Force on charge 'Q_0' at point P $= \vec{F}$

$$\vec{F} = \vec{F_{01}} + \vec{F_{02}}$$

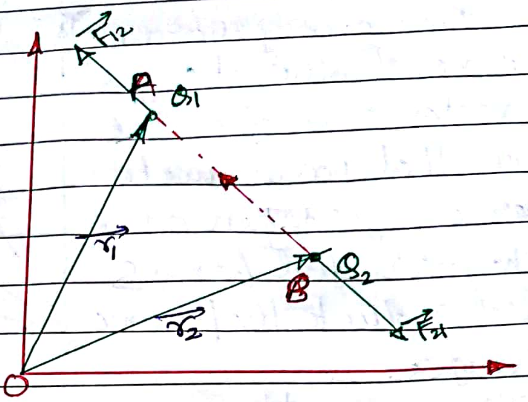
$$\vec{F} = \frac{Q_0 q_1}{4\pi\epsilon_0 x_1^3} \cdot \vec{x_1} + \frac{Q_0 q_2}{4\pi\epsilon_0 x_2^3} \cdot \vec{x_2}$$

$$\boxed{\vec{F} = \frac{Q_0}{4\pi\epsilon_0}\left[\frac{q_1}{|\vec{r_{01}}|^3} \cdot \vec{r_{01}} + \frac{q_2}{|\vec{r_{02}}|^3} \cdot \vec{r_{02}}\right]}$$

Similarly for 'n' charges.

$$\boxed{\vec{F} = \frac{Q_0}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{q_i}{|r_{0i}|^3} \cdot \vec{r_{0i}}}$$

* Coulomb force in vector form :→



Now, we know that

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 (|\vec{BA}|)^3} \cdot \vec{BA}$$

$$\boxed{\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 |\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)}$$

$$\boxed{\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 |\vec{r}_{12}|^3} \vec{r}_{12}}$$

In Δ OBA

$$\vec{OB} + \vec{BA} = \vec{OA}$$

$$\vec{r}_2 + \vec{BA} = \vec{r}_1$$

$$\therefore \boxed{\vec{BA} = (\vec{r}_1 - \vec{r}_2)}$$

$$\boxed{\vec{BA} = \vec{r}_{12}}$$

* Properties of coulomb's Law :→

(i) It is a central force, i.e. it always operates along the line joining the two charges.

(ii) Coulombian force is a long range force, it's range is upto infinity.

(iii) Force b/w neutrons & protons in a nucleus is of short range, because the force vanishes suddenly if the separation ~~Yses~~ increases beyond the distance of the order of 10^{-15} m. This force is called Yukawa force.

(ii) Coulombian force is conservative in nature (a force is called conservative where work done by it in a closed path vanishes).

* Limitations of Coulomb's Law :→

(i) Only applicable for point charge system.

Q How much is the electrostatic force stronger than the gravitational force?

Case-I) (Electron & proton combination:)

$\ominus$ —— r —— $\oplus p$ (+e)

electron proton

Electrostatic Force :-

$$F_e = \frac{k e^2}{r^2}$$

Gravitational Force :-

$$F_G = G\left(\frac{M_p m_e}{r^2}\right)$$

Now,

$$\frac{F_e}{F_G} = \frac{ke^2}{G M_p m_e} = \frac{(9\times10^{9})(1.6\times10^{-19})^2}{(6.67\times10^{-11})(1.67\times10^{-27})(9.1\times10^{-31})}$$

$$\boxed{\frac{F_e}{F_G} = 2.27\times10^{39}}$$

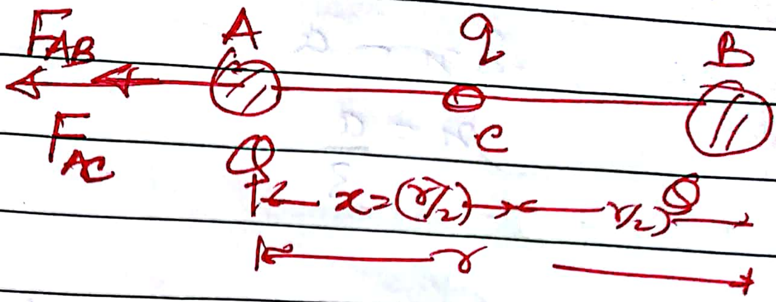
$$\boxed{F_e > F_G}$$

Case-II) (Proton – Proton combination)

(+e) (+e)

$$\boxed{\frac{F_e}{F_G} = 1.24\times10^{36}} > 1 \qquad \boxed{F_e > F_G}$$

$q = F_{qQ_2}$



$= \frac{kQq}{x^2}$

$= (r-x)^2$

$= r-x$

$r = 2x$

$\therefore x = \frac{r}{2}$ ⇒ position of q

$\cancel{\frac{kQq}{r^2}}\ 4$

$= q$

System is in equilibrium i.e. net force on any charge is zero.

$$F_{AB} + F_{AC} = 0$$

$$\frac{Q^2}{4\pi\epsilon_0 r^2} + \frac{Q.q}{4\pi\epsilon_0 \left(\frac{r}{2}\right)^2} = 0$$

$$\rightarrow \frac{Q}{4\pi\epsilon_0 r^2}\left[Q + 4q\right] = 0$$

$$\boxed{q = \left(-\frac{Q}{4}\right)}$$

$$\frac{k\,q_{eq}}{[(a+y+y')-x]^2}$$ $F_{eq} = \frac{k\,e\,q}{x^2}$ kahi kahi jake to hoga hi?

$$\frac{k\,4\,q_{eq}}{[(a+y+y')-x]^2} = \frac{k\,q_{eq}}{x^2}$$

$$2x = a+y+y'-x$$
$$3x = a+y+y'$$
$$x = \frac{a+y+y'}{3}$$

$$\frac{k\,4e\,q}{[a+y+y']^2} = \frac{k\,q\,Q}{(x)^2}$$

$$\frac{4e}{q} \times \frac{(a+y+y')^2}{} \propto \frac{1}{(a+y+y')^2}$$

$$\frac{4e}{q} = V$$

A/c to ques system is in eq^b
Net force on 'A' should be zero.

$$F_{AB} + F_{AC} = 0$$

$$\frac{(4e \times e)}{4\pi\epsilon_0 a^2} + \frac{(4e \times q)}{4\pi\epsilon_0 x^2} = 0$$

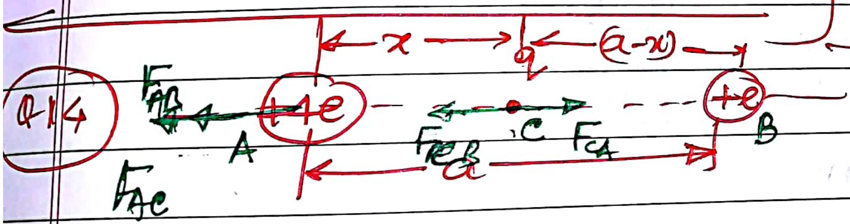
$$\frac{4e}{4\pi\epsilon_0}\left[\frac{e}{a^2} + \frac{q}{x^2}\right] = 0$$

$$\frac{e}{a^2} = \frac{-q}{\left(\frac{2a}{3}\right)^2}$$

$$q = -\frac{e}{a^2} \times \frac{4a^2}{9}$$

$$\boxed{q = -\frac{4e}{9}}$$

Q14



At eq^b
$\Sigma F_e = 0$
$F_{CA} = F_{CB}$

$$\frac{4e \cdot q}{4\pi\epsilon_0 x^2} = \frac{e \cdot q}{4\pi\epsilon_0 (a-x)^2}$$

$$\frac{4}{x^2} = \frac{1}{(a-x)^2}$$

$$\frac{2}{x} = \frac{1}{(a-x)}$$

$$2a - 2x = x$$
$$3x = 2a$$
$$\boxed{x = \frac{2a}{3}}$$

(17)

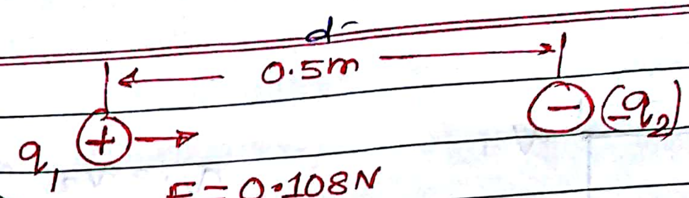


Fig-1

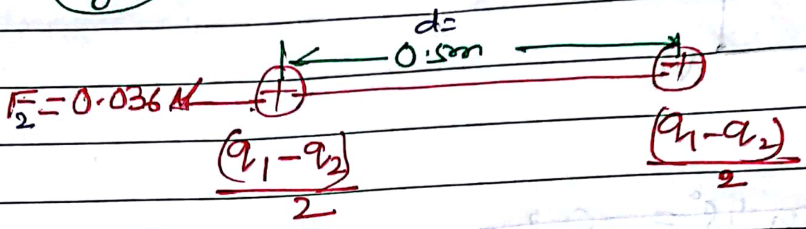


Fig-2

From fig-1

$$F_1 = \frac{q_1 q_2}{4\pi\epsilon_0 d^2}$$

$$0.108 = \frac{q_1 q_2}{4\pi\epsilon_0 d^2}$$

$$\frac{0.108}{9\times10^9} = \frac{q_1 q_2}{(0.5)^2} \quad \text{—(1)}$$

$$q_1 q_2 = \frac{0.25 \times 0.108}{9\times10^9}$$

$$\boxed{q_1 q_2 = 3\times10^{-12}\,c^2}$$

From fig-2

$$F_2 = \frac{\left(\frac{q_1-q_2}{2}\right)^2}{4\pi\epsilon_0 d^2}$$

$$0.036 = \frac{\left(\frac{q_1-q_2}{2}\right)^2}{4\pi\epsilon_0 d^2}$$

$$(q_1-q_2)^2 = 2\times10^{-6}\,c^2$$

$$(q_1+q_2)^2 = (q_1-q_2)^2 + 4q_1q_2$$

$$(q_1+q_2) = 4\times10^{-6}\,c$$

$$\boxed{q_1 = 3\times10^{-6}\,c \quad | \quad q_2 = 10^{-6}\,c}$$

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Electric field :→

* Test charge can never be -ve, it's always +ve world wide *.

→ A test charge (q) is put at the point where intensity is to be determined ~~and~~ and force (F) experienced by the charge.

$$\vec{E} = \left(\frac{\vec{F}}{q}\right)$$

For better result, $q \to 0$

$$\boxed{\vec{E} = \lim_{q \to 0} \left(\frac{\vec{F}}{q}\right)}$$

→ The ratio $\left(\frac{F}{q}\right)$ is called "intensity of the field" at that point.

→ In measurement of $\vec{E}$, we must be sure that arrival of (q) does not effect the charge distribution responsible for the original field, ∴ the ~~ch~~ (q) should be as small as possible.

→ Unit test charge or test charge is always taken positive as per assumption.

→ Direction of electric field is always taken from +ve to −ve.

→ Unit of electric field → N/C (newton per coulomb)

→ dimensions of electric field ⇒ $E = \frac{F}{q_0} = \frac{MLT^{-2}}{C} = \frac{MLT^{-2}}{A \cdot T}$

$= [MLT^{-3}A^{-1}]$

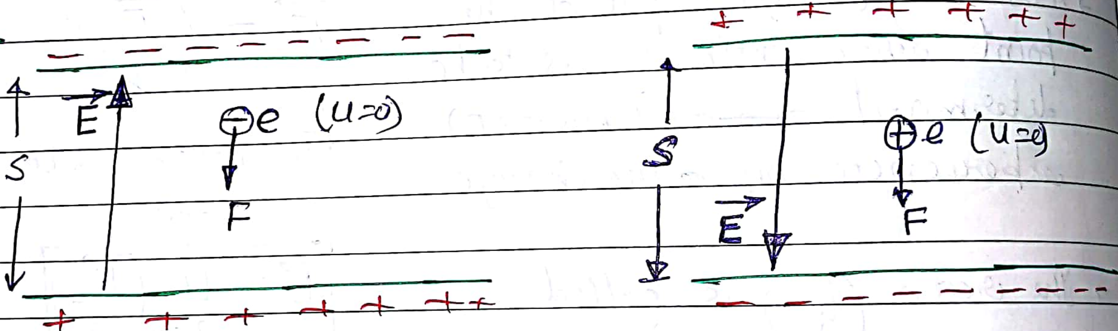
$\left(\because 1A = \frac{1C}{1s}\right)$

~~Ex 27~~

$Q = 10 \times 1.6 \times 10^{-19} C$

$Q = 1.6 \times 10^{-18} C$

NCERT
SL. Q-31



Now, Electrostatic force, $F = qE$

$Ma = qE$

$a = \left(\frac{qE}{m}\right)$

~~$V = u + at$~~

$S = ut + \frac{1}{2}at^2$

$S = 0 + \frac{1}{2}at^2$

$\boxed{t = \sqrt{\frac{2S}{a}}}$

For electr

$a_e = \frac{eE}{m_e}$

$a_p = \frac{eE}{m_p}$

$t_e = \sqrt{\frac{2 \times 1.5 \times 10^{-2} \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 2 \times 10^{4}}}$

$\boxed{t_e = 2.9 \times 10^{-9} s}$

Similarly

$\boxed{t_p = 1.25 \times 10^{-7} s}$

$\because m_p > m_e$ $a_e > a_p$

$(t_p < t_e)$

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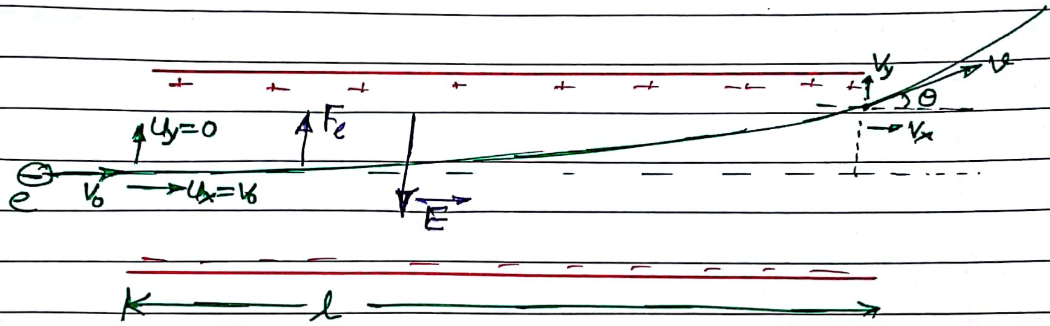
NCERT Q-33
~~(Q-34)~~



Here, $a_y = \left(\frac{eE}{m}\right)$, $a_x = 0$

Now, $V_x = (u_x + a_x t) = (V_0 + 0) = V_0$ —(I)

$V_y = (u_y + a_y t) = \left[0 + \left(\frac{eE}{m}\right)t\right] = \left(\frac{eE}{m}\right)t$ —(II)

For finding value 't'

$x = u_x t + \frac{1}{2} a_x t^2$

$l = V_0 t + \frac{1}{2}(0) t^2$

$l = V_0 \times t$

$\boxed{t = (l/V_0)}$

$\therefore V_y = \frac{eEl}{mV_0}$ —(III)

Direction,

Now $\tan\theta = \left(\frac{V_y}{V_x}\right) = \frac{eEl}{mV_0V_0}$

$\therefore \boxed{\theta = \tan^{-1}\left(\frac{eEl}{mV_0^2}\right)}$

Q-33(b)

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{V_0^2 + \left(\frac{eEl}{mV_0}\right)^2}$

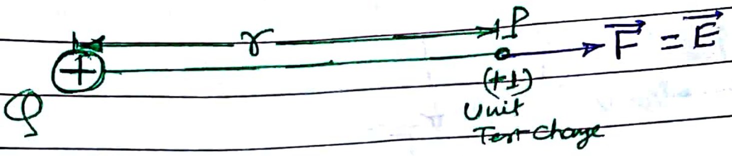
Vertical displacement

$y = u_y t + \frac{1}{2} a_y t^2$

$y = 0 + \frac{1}{2}\frac{eE}{m}\left(\frac{l}{V_0}\right)^2$

$y = \frac{1}{2}\frac{eEl^2}{...}$

① Monopole :→



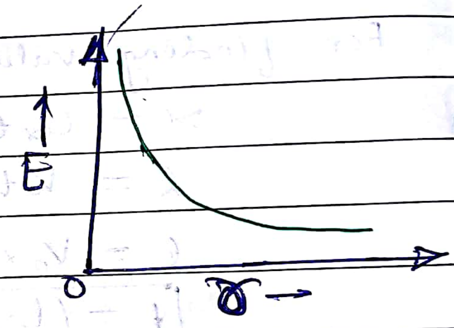
Consider a single charge 'Q' produced its field at distance 'r', then this field is known as Monop...
We have to find electric field at 'P',

$|\vec{E}|$ = Force on (+1) charge at distance 'r'

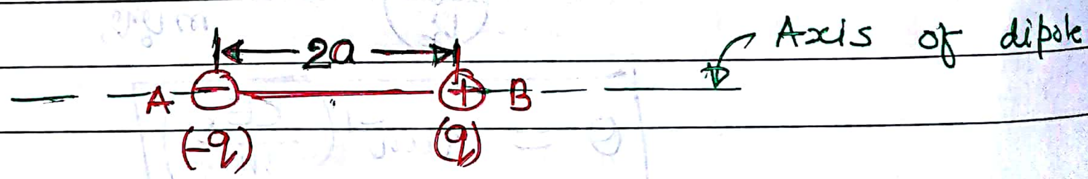
$$\vec{F} = \frac{1 \times Q}{4\pi\epsilon_0 r^2}$$

$$\therefore \boxed{E = \left(\frac{Q}{4\pi\epsilon_0 r^2}\right)}$$

$$\boxed{E \propto \frac{1}{r^2}}$$



② Dipole :→



→ when field is created by an assembly of an equal & opposite charges separated by a very small distance it is called dipole field and the system itself is called a dipole.

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→ The axis joining the two dipole is known as the ~~ax~~ "axis of dipole".

→ Direction of dipole is always taken from ~~+ve to~~ negative to positive.

→ Dipole moment :— ($\vec{p}$)

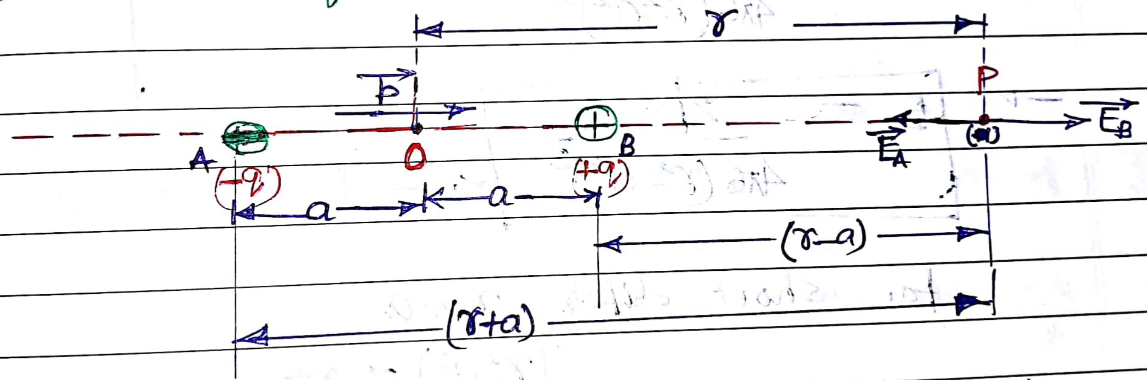
$$\boxed{|\vec{p}| = q \cdot |2\vec{a}|}$$

$$\boxed{p = q(2a)}$$

Direction of dipole alway taken frm ⊖ve to ⊕ve

Units :- Debye or (Coul - met)

5 Marks
② → ⓐ Electric field due to dipole at its axial position:-



Consider a dipole of moment $\vec{p}$ as shown in figure. We have to find electric field at point 'P' from a dipole.

From principle of superposition of field,
Resultant electricfield at point 'P',

$$\vec{E} = \vec{E}_B + (-\vec{E}_A) = (\vec{E}_B - \vec{E}_A)$$

All vectors are collinear,

$$E = (E_B - E_A)$$

$$E = (E_B - E_A)$$

$$= \frac{q}{4\pi\epsilon_0 (r-a)^2} - \frac{q}{4\pi\epsilon_0 (r+a)^2}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r+a)^2} - \frac{1}{(r+a)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{(r+a)^2 - (r-a)^2}{(r^2-a^2)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{\cancel{r^2} + \cancel{a^2} + 2ra - \cancel{r^2} - \cancel{a^2} + 2ra}{(r^2-a^2)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{4ra}{(r^2-a^2)^2}\right]$$

$$\rightarrow E = \frac{q(2a)\cdot 2r}{4\pi\epsilon_0 (r^2-a^2)^2} \qquad \because p = (q\cdot 2a)$$

$$\rightarrow \boxed{E = \frac{2p\cdot r}{4\pi\epsilon_0 (r^2-a^2)^2}} \leftarrow \text{For long dipole}$$

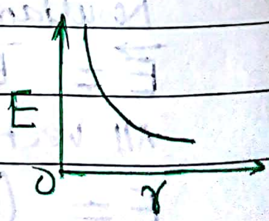
For short dipole $r >> a$

$$(r^2-a^2) \approx r^2$$

$$E = \frac{2p\cdot \cancel{r}}{4\pi\epsilon_0 r^{\cancel{4}\,3}}$$

$$\therefore \boxed{E = \frac{2p}{4\pi\epsilon_0 r^3}}$$

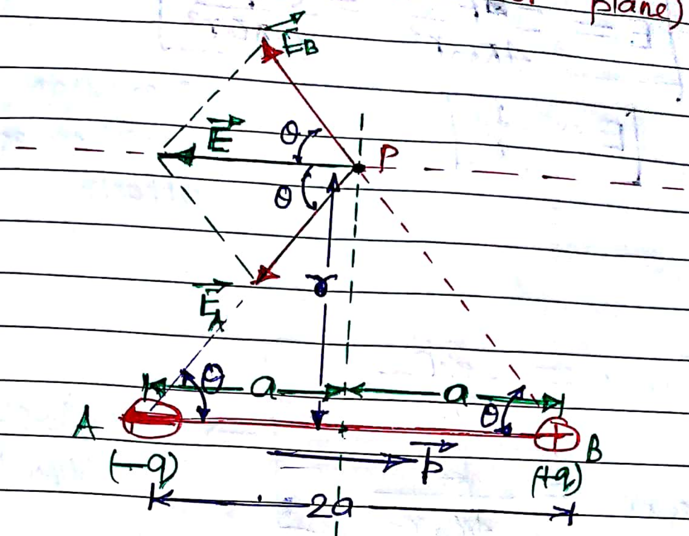
$$\boxed{E \propto \frac{1}{r^3}}$$



In vector for

$$\boxed{\vec{E} = 2\vec{p}}$$

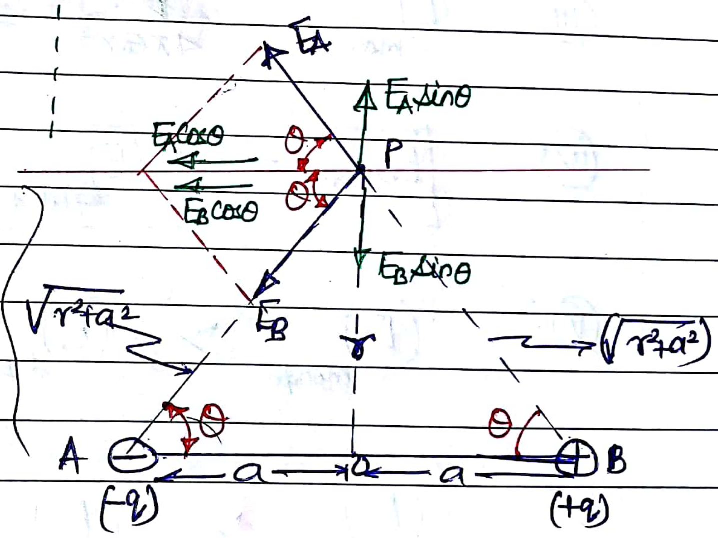
5 Marks 2(b) **Electric field due to dipole at equitorial position :→ or (Bisector plane)**



$E_A = E_B = E'$

$= \dfrac{q}{4\pi\epsilon_0(\sqrt{r^2+a^2})^2}$

$= \dfrac{q}{4\pi\epsilon_0(r^2+a^2)}$



Let E be resultant Electric field.

x-component :-

$$E_x = (-E_A\cos\theta) + (-E_B\cos\theta) = (-2E'\cos\theta)$$

$$E_y = (E_A\sin\theta - E_B\sin\theta) = (E'\sin\theta - E'\sin\theta) = 0$$

∴ Resultant intensity,

$$E = -2E'\cos\theta = -2\frac{q}{4\pi\epsilon_0(r^2+a^2)} \times \frac{a}{\sqrt{r^2+a^2}}$$

$$E = \frac{-p}{4\pi\epsilon_0(r^2+a^2)^{3/2}}$$

$$\therefore \boxed{\vec{E} = \frac{-\vec{p}}{4\pi\epsilon_0(r^2+a^2)^{3/2}}}$$ ← For long dipole

For short dipole, $(r >> a)$ i.e. $(r^2 + a^2)$

$$\boxed{\vec{E} = \frac{-\vec{p}}{4\pi\epsilon_0 r^3} = \frac{(-)\vec{p}}{4\pi\epsilon_0 r^3}}$$

$$\boxed{E \propto \frac{1}{r^3}}$$

(-)ve sign denotes the direction of $\vec{E}$ & $\vec{p}$ is opposite.

Notes:→

(I) $E_{axial} = \frac{2p}{4\pi\epsilon_0 r^3}$

(II) $E_{equat} = \frac{p}{4\pi\epsilon_0 r^3}$

$\Rightarrow (E)_{dipole} \propto \frac{1}{r^3}$

(III) $(E)_{mon} = \frac{q}{4\pi\epsilon_0 r^2} \Rightarrow (E)_{monop} \propto \frac{1}{r^2}$

(IV) $\boxed{E_{axial} = 2\,(E)_{equatorial}}$

(V) $(E)_{monopole} > (E)_{dipole}$

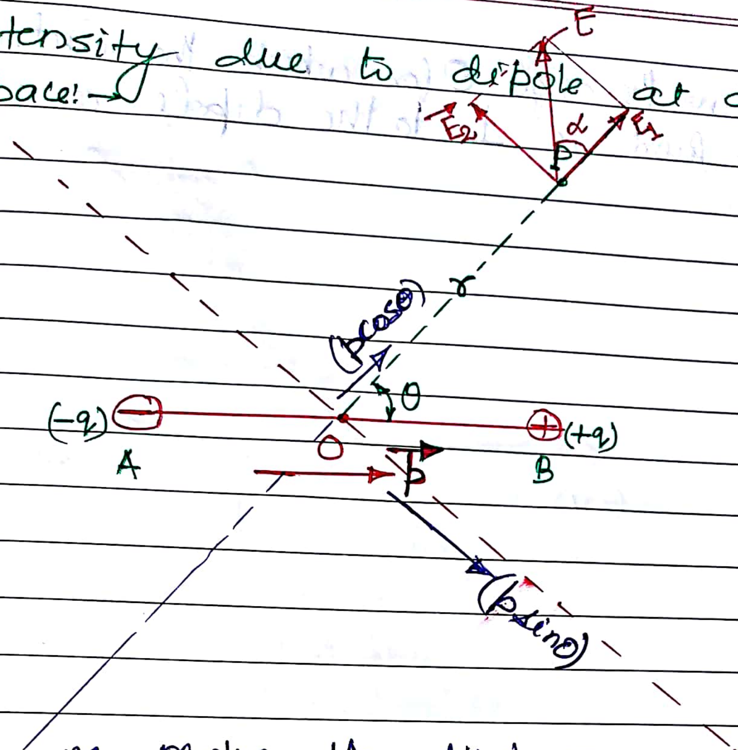
(VI) $(E)_{axial} > (E)_{equatorial}$

* Intensity at any point due to dipole :-

competitive register

(Electrostatics)

* 2(c) Intensity due to dipole at any point in space:-



Here, we resolve the dipole moment $\vec{p}$ in two ⊥ component

E_1 = Electricfield due to dipole component $(p\cos\theta)$

E_2 = Electricfield due to dipole component $(p\sin\theta)$

$$E_1 = \frac{2p\cos\theta}{4\pi\epsilon_0 r^3}$$

& $$E_2 = \frac{p\sin\theta}{4\pi\epsilon_0 r^3}$$

Here E_1 & E_2 are perpendicular to each other.

$$E = \sqrt{E_1^2 + E_2^2} = \sqrt{\frac{4p^2\cos^2\theta}{(4\pi\epsilon_0 r^3)^2} + \frac{p^2\sin^2\theta}{(4\pi\epsilon_0 r^3)^2}}$$

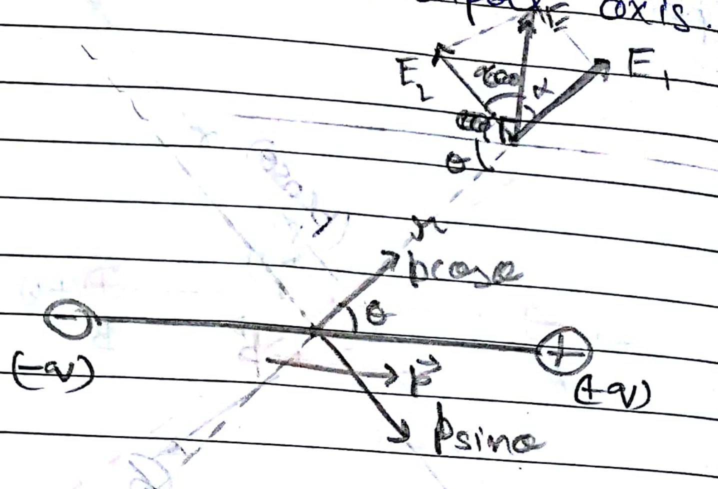
$$E = \frac{p}{4\pi\epsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$\boxed{E = \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + 1}}$$

Direction :- α = Angle betn $\vec{E_1}$ & $\vec{E}$

$\tan\alpha = \left(\frac{E_2}{E_1}\right) = \left(\frac{\tan\theta}{2}\right)$ $\therefore$ $\alpha = \tan^{-1}\left\{\frac{1}{2}\tan\theta\right\}$

Q/- Determine the angle θ for which the electric intensity due to field is $\perp$ to the dipole axis.



$$E_1 = \frac{2p\cos\theta}{4\pi\epsilon_0 (r)^3}$$

$$E_2 = \frac{p\sin\theta}{4\pi\epsilon_0 (r)^3}$$

$\theta + 90° + \alpha = 180$

$\alpha = 90°$

$\tan 90° = \frac{E_2}{E_1}$

$\theta + 90° + \alpha = 180°$

$\theta + \alpha = 90$

$\alpha = 90 - \theta$

$\tan\alpha = \frac{E_2}{E_1}$

$\tan(90-\theta) = \frac{E_2}{E_1}$

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$$\cot\theta = \frac{E_2}{E_1}$$

$$\cot\theta = \frac{\cancel{p}\sin\theta}{\cancel{4\pi\epsilon_0(r)^3}} \times \frac{\cancel{4\pi\epsilon_0(r)^3}}{2\cancel{p}\cos\theta}$$

$$\cot\theta = \frac{1}{2}\tan\theta$$

$$\frac{1}{\tan\theta} = \frac{\tan\theta}{2}$$

$$2 = \tan^2\theta$$

$$\tan\theta = \pm\sqrt{2}$$

$$\theta = \tan^{-1}(\sqrt{2})$$

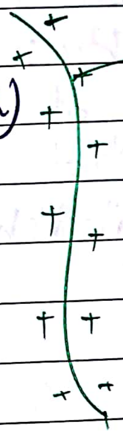
Pg. 1.32
Q 41, 42, 38, 39, 40
Problems for practice
Q 8, 9, 10

* Continuous charge distribution :→

ⓐ Linear charge distribution :→
(λ)

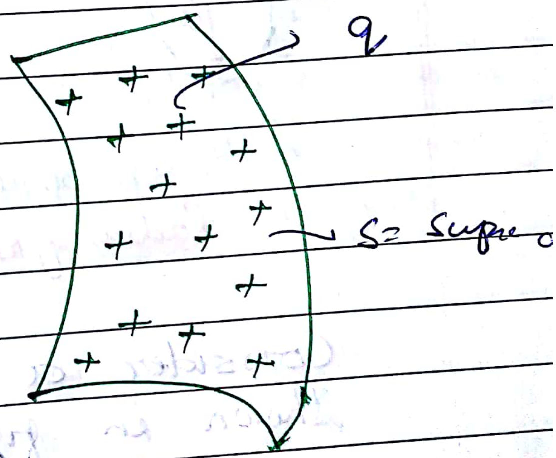
λ = (Charge per unit length)

$$\boxed{\lambda = \left(\frac{q}{l}\right)}$$



ⓑ Surface charge density : (σ)

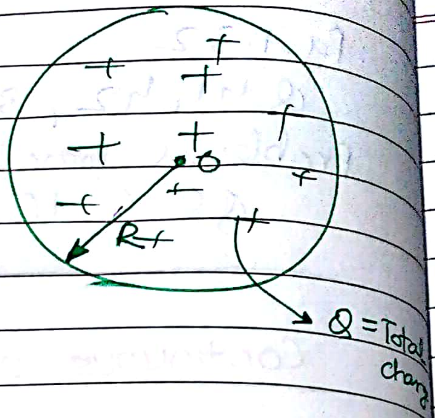
$$\boxed{\sigma = \frac{q}{S}}$$



(c) Volume charge density (ρ):→

$$\boxed{\rho = \frac{Q}{\frac{4}{3}\pi R^3}}$$

$$\boxed{\rho = \frac{(\text{Charge})}{(\text{Volume})}}$$



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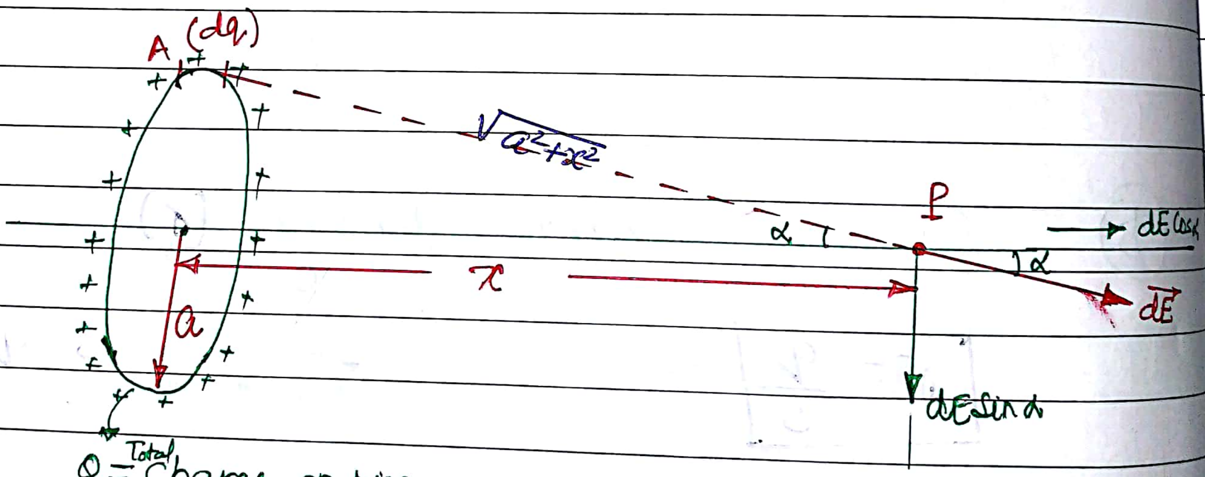
Q – A charge is distributed uniformly over a ring of radius 'a'. Obtain an expression for the electric intensity E at a point on the axis of the ring. Hence show that for points at large distances from the ring, it behaves like a point charge.



Q = Total Charge on ring
a = Radius of ring

Consider a charged ring of radius 'a' as shown in figure. Total charge on ring 'Q'. Assume an elementry charge 'dq' at point 'A' as shown in figure.

$1 + \frac{1}{2} = \frac{3}{2}$

By symmetry,

$$\oint dE \sin\alpha = 0$$

∴ Total ~~charge~~ Electric field due to charge ring,

$$E = \oint dE \cos\alpha$$

$$= \oint \frac{dq}{4\pi\epsilon_0 (\sqrt{a^2+x^2})^2} \cdot \frac{x}{(\sqrt{a^2+x^2})}$$

$$= \frac{x}{4\pi\epsilon_0 (a^2+x^2)^{3/2}} \oint dq$$

$$\therefore \boxed{E = \frac{Qx}{4\pi\epsilon_0 (a^2+x^2)^{3/2}}}$$

Prove that for a large distance 'x' it behaves as point charge.

$$E = \frac{Q \cdot \cancel{x}}{4\pi\epsilon_0 x^{\cancel{3}\,2} \left[\frac{a^2}{x^2}+1\right]^{3/2}}$$

$$E = \frac{Q}{4\pi\epsilon_0 x^2 \left(\frac{a^2}{x^2}+1\right)^{3/2}}$$

Put $x >> a$ $\quad \frac{a^2}{x^2} \simeq 0$

$$\therefore \boxed{E = \frac{Q}{4\pi\epsilon_0 x^2}} \longleftarrow$$

It behaves a point charge when $x >> a$

Note:→ (i) 'E' will be max^m

$$\left(\frac{dE}{dx}\right) = 0 \quad , \quad \frac{Q}{4\pi\epsilon_0} \frac{d\left[\frac{x}{(a^2+x^2)^{3/2}}\right]}{dx} = 0$$

$$\boxed{x = \pm \frac{a}{\sqrt{2}}}$$

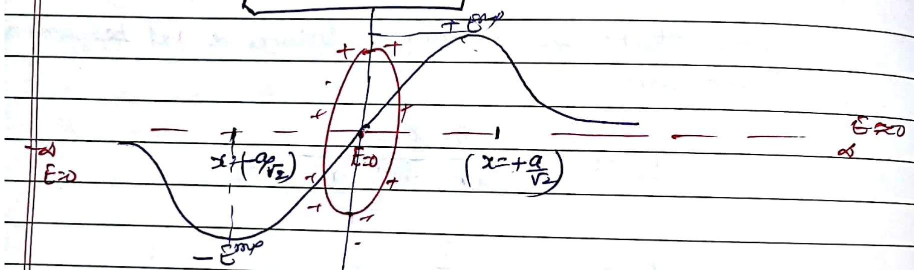
$$\frac{q}{4\pi\epsilon_0}\frac{d\left[\frac{x}{(x^2+a^2)^{3/2}}\right]}{dx}=0$$

$$\frac{(x^2+a^2)^{3/2}\cdot 1 - x\cdot\frac{3}{2}(x^2+a^2)^{\frac{1}{2}}\cdot 2x}{\left[(x^2+a^2)^{3/2}\right]^2}=0$$

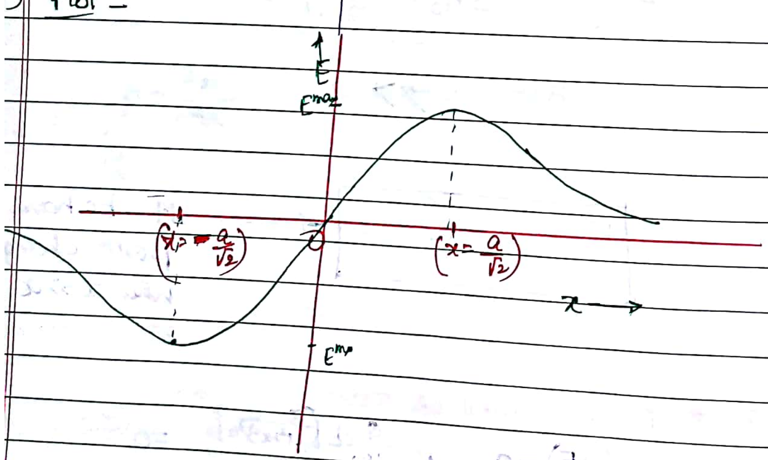
$\rightarrow (x^2+a^2)^{\frac{1}{2}}\left[x^2+a^2-3x^2\right]=0$

$\rightarrow \quad -2x^2+a^2=0$

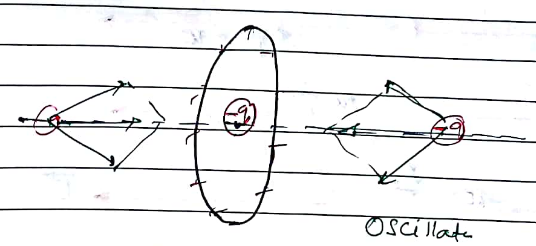
$\rightarrow \quad \boxed{x=\pm\frac{a}{\sqrt{2}}}$



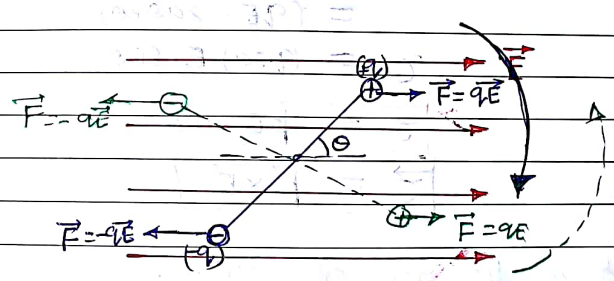
Plot –



(11) If a charge $(-q)$ is slightly displaced from centre of ring then the motion of charge is oscillatory.



* Behaviour of dipole in uniform electric field :→

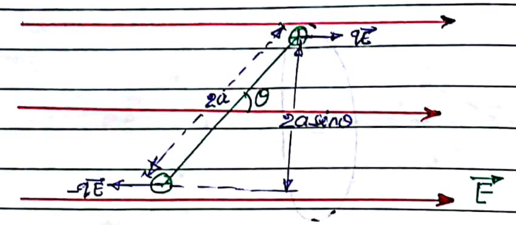


In uniform electric field,

→ Net force along electric field (Horizontal) is zero. Hence, no translatory motion

→ Due to couple only oscillatory motion.

* Torque on dipole in uniform electric field :→



Consider a dipole placed in a uniform field as shown in figure. If it ~~will~~ rotate at an angle θ with $\vec{E}$ then a couple will be acted.

Torque on dipole $\tau = F \cdot (\perp^r \text{ distance})$

$= (qE \cdot 2a\sin\theta)$

$\tau = (q \cdot 2a)\, E \sin\theta$

$$\boxed{\tau = pE\sin\theta}$$

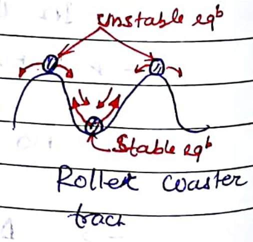
$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

→ Equilibrium position of dipole :→

$\tau = 0$

$PE \sin\theta = 0$

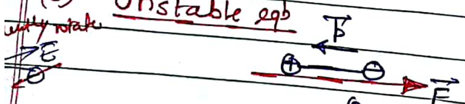
$\therefore \boxed{\theta = 0^\circ \text{ or } 180^\circ}$



a) Stable eqb

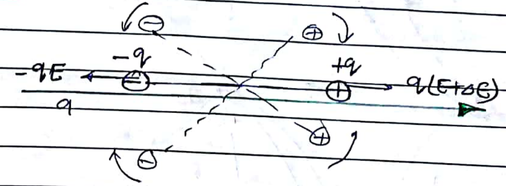
$(\theta = 0^\circ)$

(b) Unstable eqb



$(\theta = 180^\circ)$

* Behaviour of a dipole in non-uniform field:→



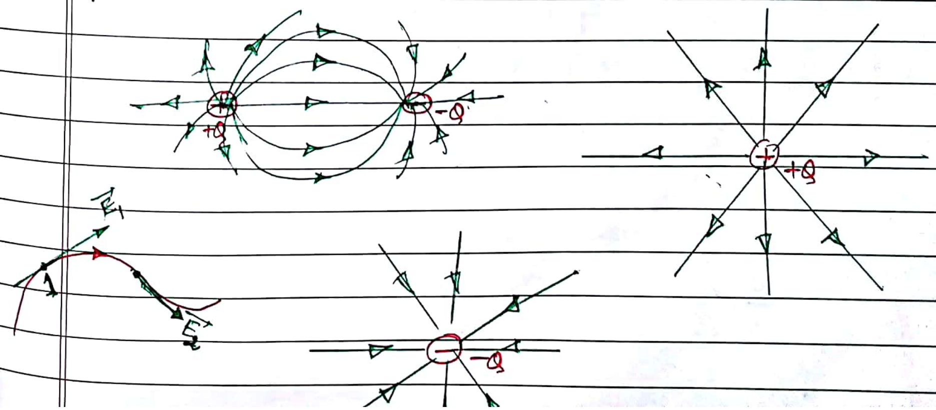
Net force on dipole in uniform electric field,

$$\boxed{F_x = p \cdot \frac{dE}{dx}}$$

Hence, it has both translatory as well as oscillatory motion in non-uniform electric field.

* Line of force :→

It is a hypothetical curve drawn in an electric field in such a way, that the tangent to the curve at any point determine the direction of intensity at that point.



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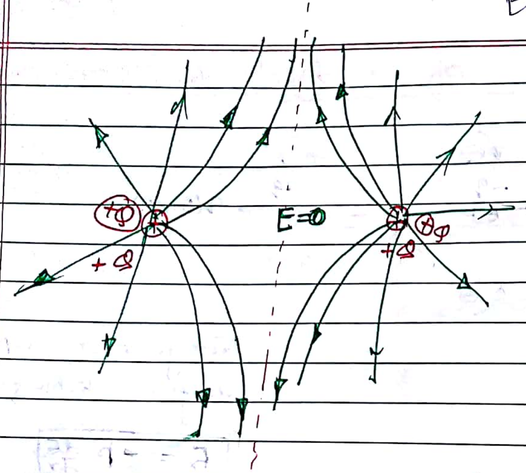
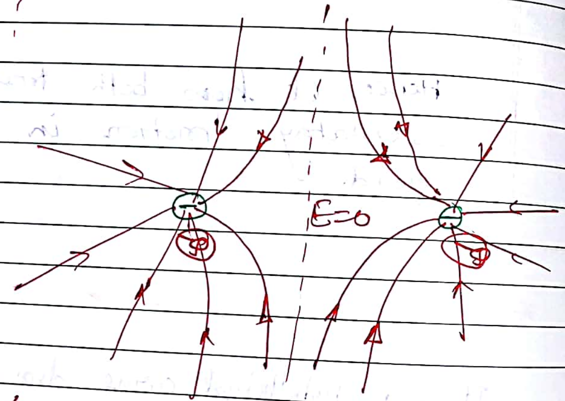
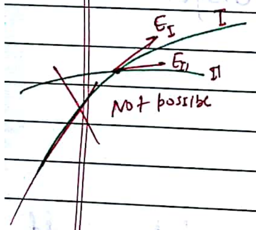
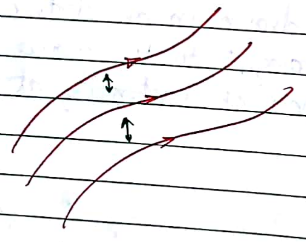
Electric field line of charged ring?



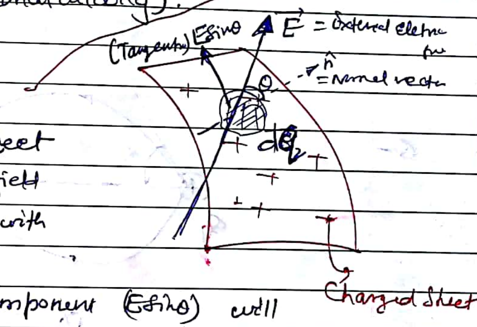






- → Line of force always starts from positive charge & gets terminated at negative charge or travel to infinity.
- → No two lines of force can intersect because if they intersect at a point then two tangents can be drawn at the same time which is not possible.
- → When a number of lines of force travel in space they repel each other.
- → It follows "inverse square law".
- → A line of force always meets the surface of a conductor at 90°C (perpendicularly).

→ Proof :-



Consider a charged sheet placed in uniform electric field which makes an angle 'θ' with normal to the field.

Now, a tangential component (E sinθ) will be acted on sheet. Due to tangential component (E sinθ) a force dq (E sinθ) will act on sheet of elementary area 'ds' charge.

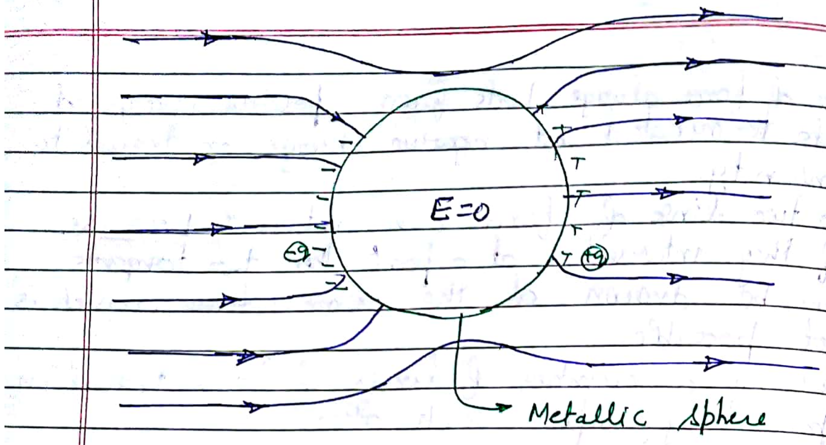
A/c to electrostatic, total force on charge is zero ?

$\therefore dq_0 (E \sin\theta) = 0$

$\sin\theta = 0$

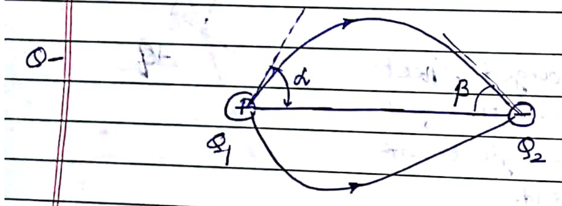
$\therefore \boxed{\theta = 0°}$

Hence, $\vec{E}$ will always intersect at 90° to metallic surface.



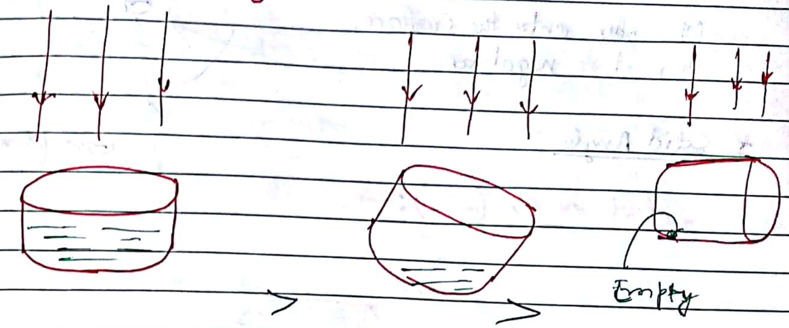
→ Number of field lines ∝ magnitude of charge
∝ strength of electric field.

Q-

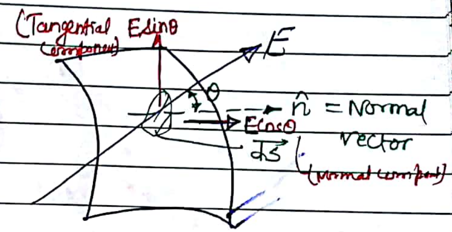


$$\boxed{\frac{\alpha}{\beta} = \frac{Q_2}{Q_1}}$$

* Electrostatic flux :→



A/c to Faraday's.



$d\phi = (E\cos\theta) \cdot ds$

$\boxed{d\phi = E\,ds\cos\theta}$

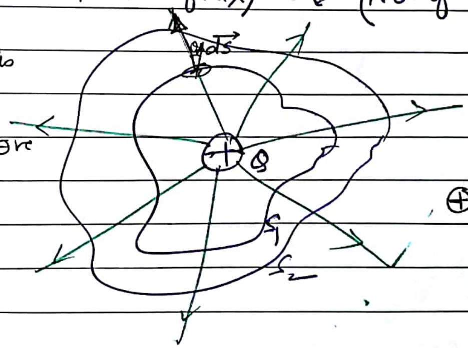
Total flux, $\boxed{\phi = \oint E\,ds\cos\theta}$

$\boxed{\phi = \oint \vec{E} \cdot \vec{ds}}$

$\boxed{\Delta\phi = \vec{E} \cdot \vec{\Delta s}}$ $\boxed{\phi = \vec{E} \cdot \vec{S}}$

(Electrostatic flux) ∝ (No. of electric field lines)

Electrostatic flux is scalar quantity.
It may be ⊕ve, ⊖ve or zero.



$0 < \theta < 90°$

⊕ve flux

When exit from volume then ⊕ve flux

$\boxed{\phi_1 = \phi_2}$

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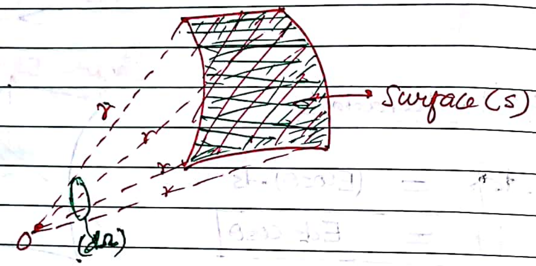
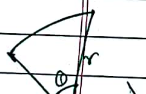
When flux enter the surface, then it is negative.

* Solid Angle:

Solid angle (Ω) :→



$$\text{Solid angle}(\Omega) = \left(\frac{\text{Surface area}}{(\text{Radius})^2}\right)$$



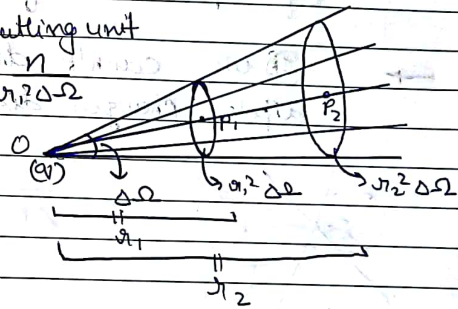
$$\theta = \left(\frac{\text{Arc length}}{\text{Radius}}\right)$$

$$\boxed{\Omega^{max} = 720° = 4\pi}$$

Q/ Ng.1.42 Show that the $1/r^2$ dependence of electric field of a point charge is consistent with the concept of the electric field lines.

∴ no. of lines of force cutting unit area element at $P_1 = \frac{n}{r_1^2 \Delta\Omega}$

no. of lines of force cutting unit area element at $P_2 = \frac{n}{r_2^2 \Delta\Omega}$

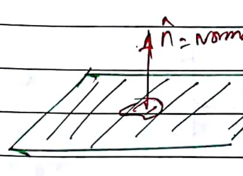


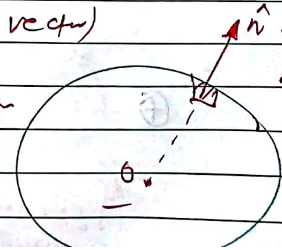
As electric field strength ∝ Density of lines of force.

$$\therefore \frac{E_1}{E_2} = \frac{\cancel{n}}{r_1^2 \cancel{\Delta\Omega}} \cdot \frac{r_2^2 \cancel{\Delta\Omega}}{\cancel{n}} = \frac{r_2^2}{r_1^2}$$

$$\therefore \boxed{E \propto \frac{1}{r^2}}$$

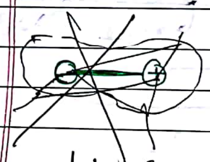
Normal Area :→ (Area vector)



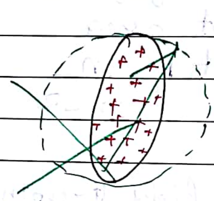


* Gaussian Surface :-

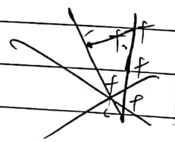
- It is a closed surface.
- Symmetric about the charges.
- Electric field lines are always normal to the Gaussian surface.
- Magnitude of Electric field lines is always constant on Gaussian surface.
- It is only applicable for infinity long charged wire, infinitely large charged plane sheet or spherical surfaces.
- We cann't draw Gaussian surface for dipole, charged disc or etc.

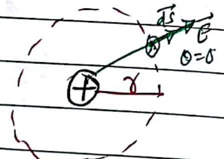


We Cannt draw

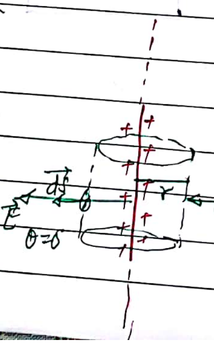


- It is not applicable for finite length of wire or finite charge sheet.





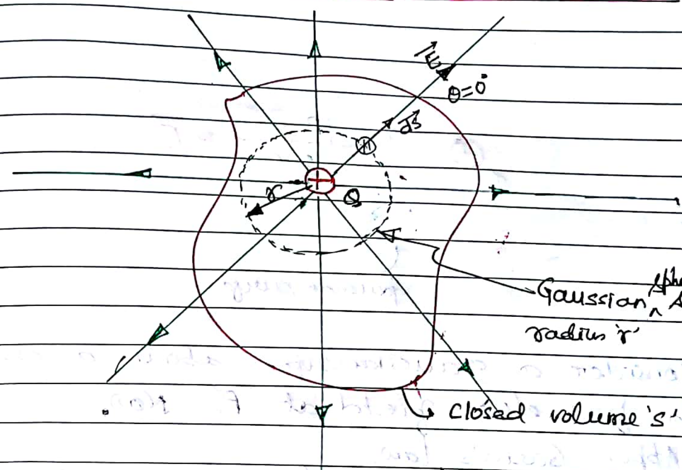
Spherical Gaussian Surface



Cylindrical Gaussian Surface.

3 Marks

* Gauss's Law or Gauss's Theorem :-



Gaussian spherical surface of radius 'r'

closed volume 's'

Consider a charge 'Q' in enclosed. volume 's' as shown in figure. we have to find flux through closed surface 's'. Take an ~~eten~~ Gaussian spherical surface of radius 'r' as shown in figure.

Now, flux through Gaussian surface = Flux through closed surface 's'

Flux through closed surface 's' = Flux through Gauss's surface

$$\phi = \oint E \cdot ds \cos 0$$

$$\phi = E \oint ds$$

$$\phi = \frac{Q}{\cancel{4\pi\epsilon_0 r^2}} \times \cancel{4\pi r^2}$$

$$\therefore \boxed{\phi = \frac{Q}{\epsilon_0}}$$

The thm. states that the total electric flux passing through a closed surface is given by $\frac{1}{\epsilon_0}$ times the total charge enclosed.

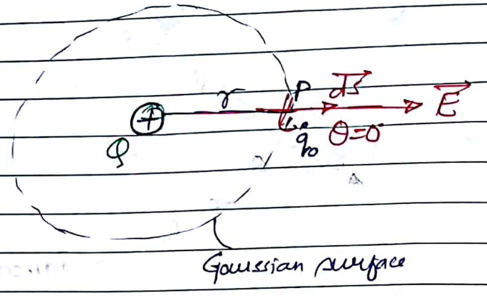
Note: -

$\cdot q_1 \quad \cdot (-q_2) \quad \cdot q_3 \quad \cdot (q_4) \quad \cdot q_6 \quad \cdot q_7$

$$\phi = \frac{\text{Charge enclosed}}{\epsilon_0} = \frac{q_1 + (-q_2) + (q_3) + (q_4)}{\epsilon_0}$$

Q. Deduce coulomb's law from Gauss's thm. :-

Pg. 1.50



Consider a gaussian surface about a charge q to find electric field at 'P'. Now,

Apply Gauss's law,

$$\phi = \oint \vec{E}\cdot\vec{ds} = E\oint ds$$

$$\frac{Q}{\epsilon_0} = E \times 4\pi r^2$$

$$\therefore \boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$$

Put 'q_0' just outside the Gaussian surface

$\therefore$ Force on q_0 = $q_0 E$

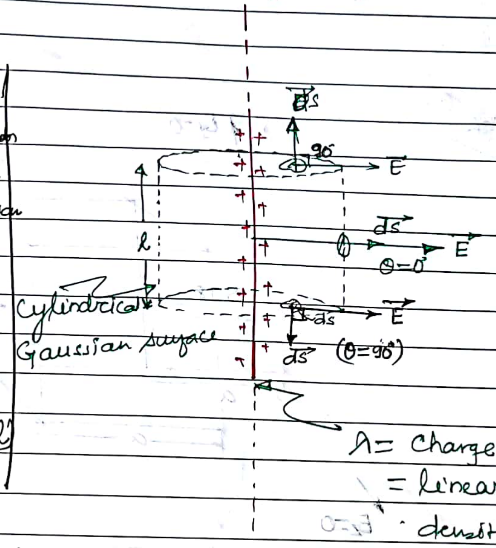
$$\therefore \boxed{F = \frac{q_0 Q}{4\pi\epsilon_0 r^2}}$$

Application of Gauss's law :→

3 Marks

a) Electric field due to infinitely long charged ~~line~~ wire :-

Consider an infinitely long uniformly charged distribution wire. Now consider a cylindrical Gaussian surface of radius 'r' and length 'l' as shown in figure.



Charge enclosed = q
= (λl)

~~Consider~~

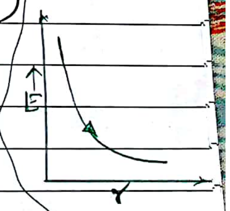
λ = Charge / length
= linear charge density

Apply Gauss's law,

$$\oint \vec{E}\cdot\vec{ds} = \frac{(q)_{encl}}{\epsilon_0}$$

$$\int_{curv\,surf} E\,ds\cos 0 + \int_{top} E\,ds\cos 90 + \int_{Bottom} E\,ds\cos 90 = \left(\frac{\lambda l}{\epsilon_0}\right)$$

$$\rightarrow E\int_{curv\,sur} ds + 0 + 0 = \left(\frac{\lambda l}{\epsilon_0}\right)$$



$$\rightarrow E \times 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

or

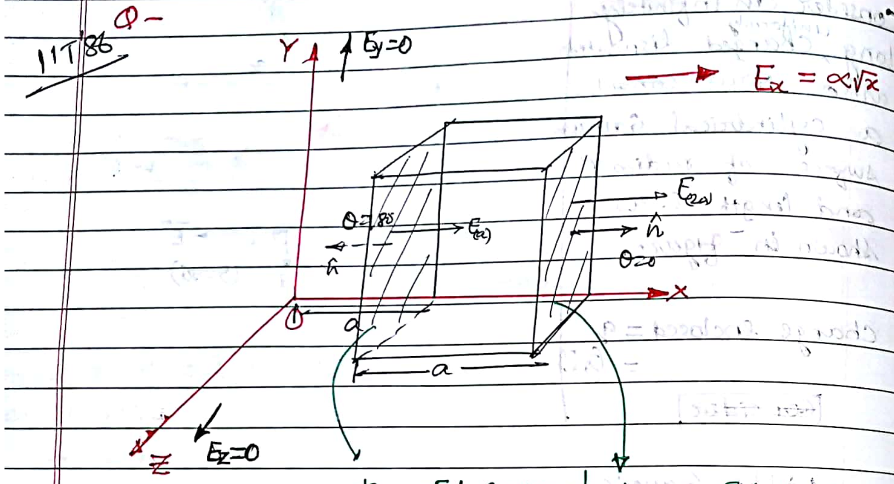
$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$$

$$\therefore \boxed{E \propto \frac{1}{r}}$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\cdot\hat{r}}$$

$\hat{r}$ = unit vector along radius

11T'86 Q -



$\phi_L = EA\cos 180$
$= -\alpha\sqrt{a}\cdot a^2$
$\phi_L = (-)\alpha a^{5/2}$

$\phi_R = EA\cos 0°$
$= \alpha\sqrt{2a}\cdot a^2$
$\phi_R = \sqrt{2}\,\alpha a^{5/2}$

$\therefore$ Total flux $\phi = (\phi_L + \phi_R)$

$$\boxed{\phi = \alpha a^{5/2}(\sqrt{2}-1)}$$

From Gauss' Law

$$\phi = \frac{q}{\epsilon_0}$$

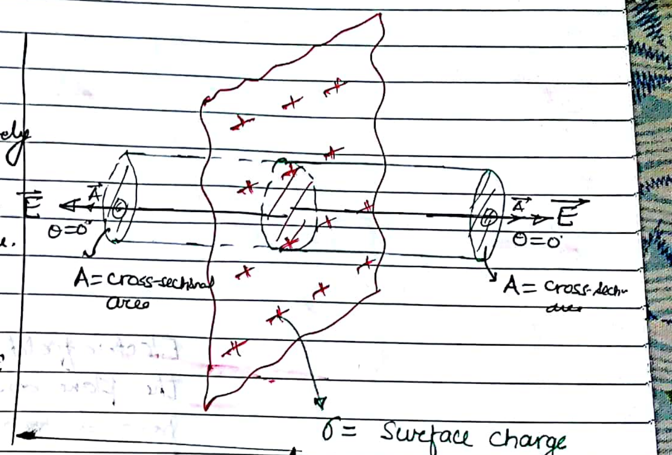
$$q = \epsilon_0 \phi$$

$$\boxed{q = \epsilon_0 \alpha a^{5/2}(\sqrt{2}-1)}$$

2 Marks (b)

Electric field due to infinitely large charged plane :-

Consider an infinitely large plane sheet of surface charge density 'σ' as shown in figure.
Now, draw a cylindrical Gaussian surface of cross-sectional area 'A'.



σ = Surface charge density
= (Charge/Area)

From Gauss's law,

$$\oint \vec{E}\cdot d\vec{s} = \frac{(q)_{enc}}{\epsilon_0}$$

$$\boxed{\cancel{\oint E\,ds\cos 0° = \frac{(\sigma A + \sigma A)}{\epsilon_0}}}$$

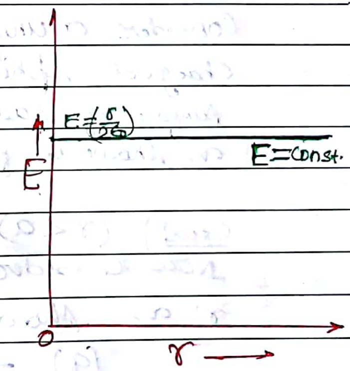
$$(EA\cos 0 + EA\cos 0) = \frac{\sigma A}{\epsilon_0}$$

$$2E\cancel{A} = \left(\frac{\sigma \cancel{A}}{\epsilon_0}\right)$$

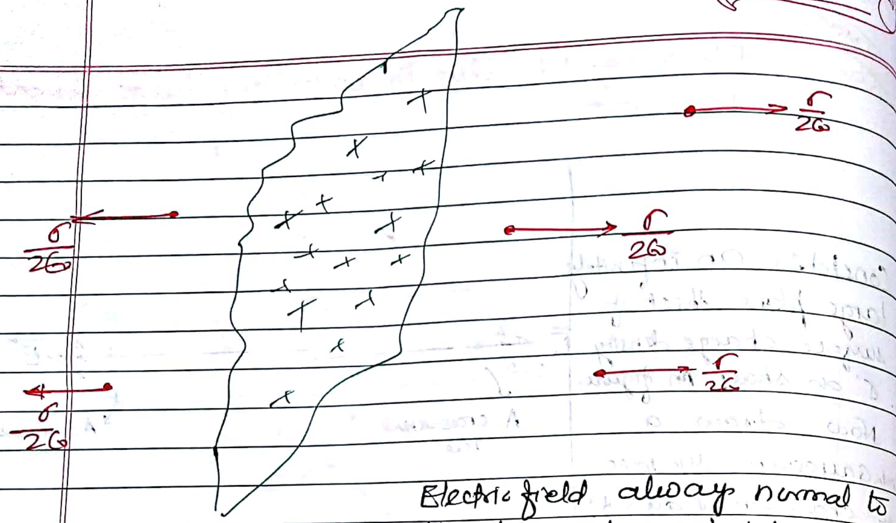
$$\therefore \boxed{E = \frac{\sigma}{2\epsilon_0}}$$

In vector form,

$$\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}\cdot\hat{n}}$$



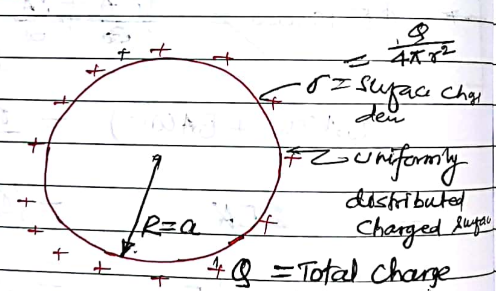
E = Const.



Electric field always normal to the plane and is independent from distance.

(C) Electric field due to uniformly charged Spherical Shell / solid Sphere :→

Consider a uniformly charged spherical surface of radius 'R' as shown in figure.



Case I) $(r < a)$

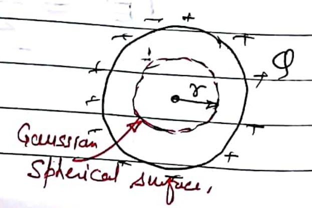
Now consider a spherical Gaussian surface of radius 'r' as shown in fig.

$(q)_{enc} = 0$

From Gauss's law

$\oint \vec{E} \cdot \vec{ds} = \frac{0}{\epsilon_0}$

$\therefore \boxed{E_{in} = 0}$



Case-II) $r > a$

$(q)_{enc} = Q$

From Gauss's law,

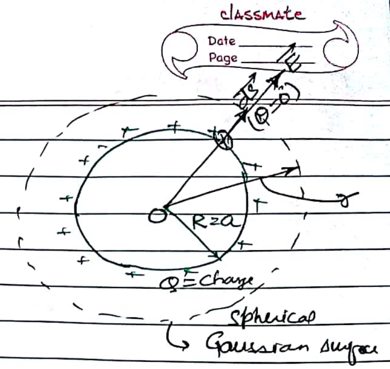
$\oint \vec{E} \cdot \vec{ds} = \frac{Q}{\epsilon_0}$

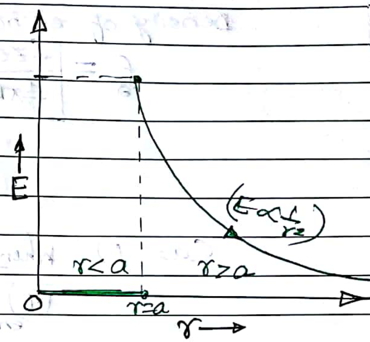
$\rightarrow E \oint ds = \frac{Q}{\epsilon_0}$

$\rightarrow E \times 4\pi r^2 = \frac{Q}{\epsilon_0}$

$\therefore \boxed{E_{out} = \frac{Q}{4\pi\epsilon_0 r^2}}$

$\boxed{E \propto \frac{1}{r^2}}$



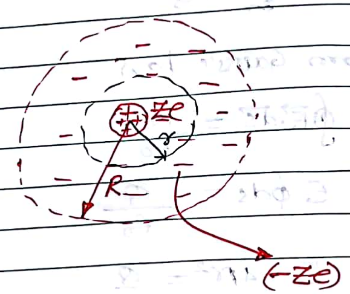


Case III) when $r = a$

$\boxed{E_{surface} = \frac{Q}{4\pi\epsilon_0 a^2}}$

—o—

NCERT Page 40 (Q- 1.13)



Density of electron

$$\rho_0 = \boxed{\frac{-Ze}{\frac{4}{3}\pi R^3}}$$

Size of nucleus is negligible but not charge.

Case-I) When $r < R$

$$(q)_{enc} = Ze + \left(\rho_0 \times \frac{4}{3}\pi r^3\right)$$

$$(q)_{enc} = Ze + \left[\frac{-Ze}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3\right]$$

$$(q)_{enc} = Ze\left[1 - \frac{r^3}{R^3}\right]$$

From Gauss's law:-

$$\oint \vec{E}\cdot\vec{ds} = \frac{(q)_{enc}}{\epsilon_0}$$

$$\rightarrow E \times 4\pi r^2 = \frac{Ze\left(1 - r^3/R^3\right)}{\epsilon_0}$$

$$\rightarrow \boxed{E_{in} = \frac{Ze}{4\pi\epsilon_0}\left[\frac{1}{r^2} - \frac{r}{R^3}\right]}$$

($r<R$)

Case II) $r > R$

$$(q)_{en} = Ze + (-Ze) = 0$$

From Gauss's law

$$\oint \vec{E}\cdot\vec{ds} = \frac{0}{\epsilon_0}$$

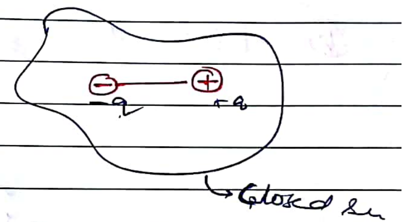
$$\therefore \boxed{E_{out} = 0} \quad (r > R)$$

* Application of Gauss's law

(I) Dipole inside a enclosed surface:

$$\phi = \frac{q + (-q)}{\epsilon_0}$$

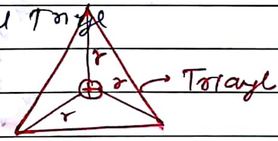
$$\boxed{\phi = 0}$$



(II) Charge of centre of equilateral Tringle

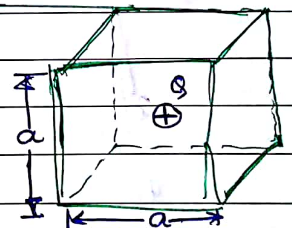
$$\boxed{\phi = 0}$$

Because it is not a closed surface.



(III) Charge on a Centre of ~~close~~ cube:→

$$\boxed{\phi = \frac{q}{\epsilon_0}}$$



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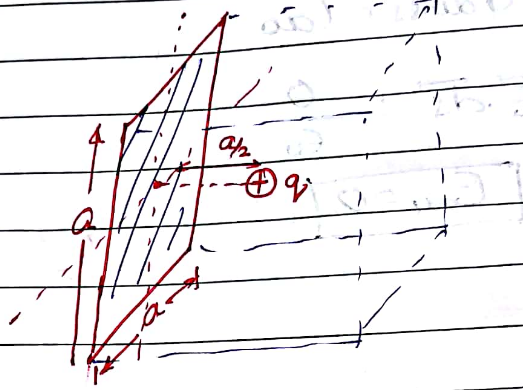
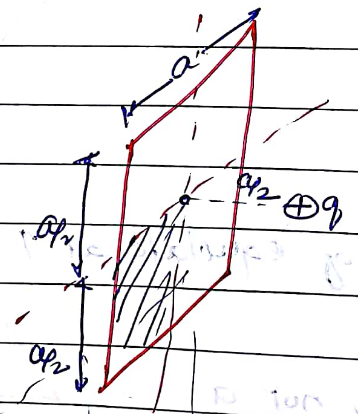
(IV) Flux through a square and charge placed as shown in fig.



Flux through shaded portion

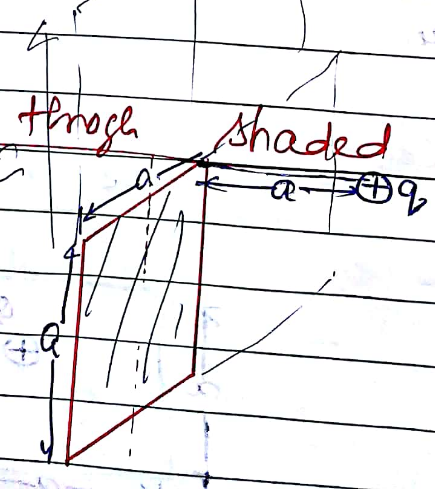
$$\boxed{\phi = \frac{1}{6}\left(\frac{q}{\epsilon_0}\right)}$$

(V) Flux through shaded portion:→



$$\boxed{\phi = \frac{1}{24}\left(\frac{q}{\epsilon_0}\right)}$$

(VI) Flux through shaded portion:-

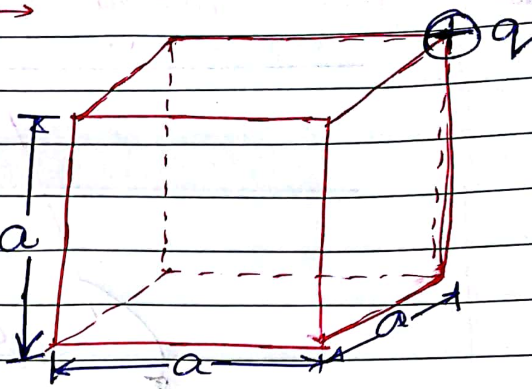


$$\boxed{\phi = \frac{1}{24}\left(\frac{q}{\epsilon_0}\right)}$$

Page

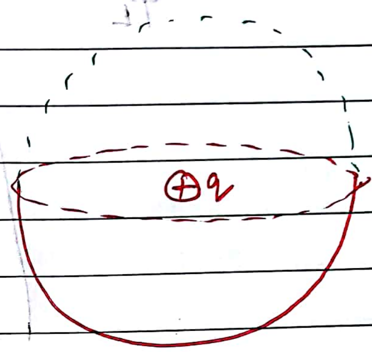
(VII) Flux through cube:→

$$\boxed{\phi = \frac{1}{8}\left(\frac{q}{\epsilon_0}\right)}$$

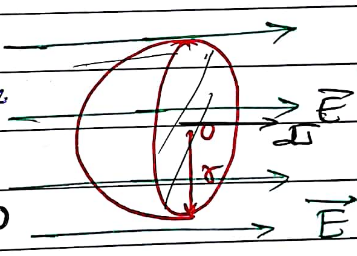


(VIII) Flux through curve surface

$$\boxed{\phi = \frac{1}{2}\left(\frac{q}{\epsilon_0}\right)}$$



(IX) Flux through curve surface



$$\phi = (\phi_{curve} + \phi_{circle}) = 0$$

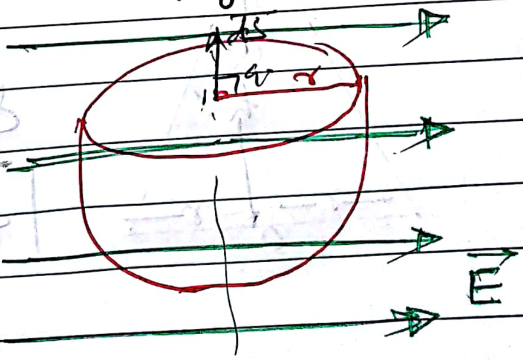
$$\phi_{curve} = -\phi_{circle}$$

$$= -EA \cdot \cos 0$$

$$= (-E\pi r^2)$$

Note:- Flux enter through curve surface is same as flux exit from circular surface - ve net flux zero.

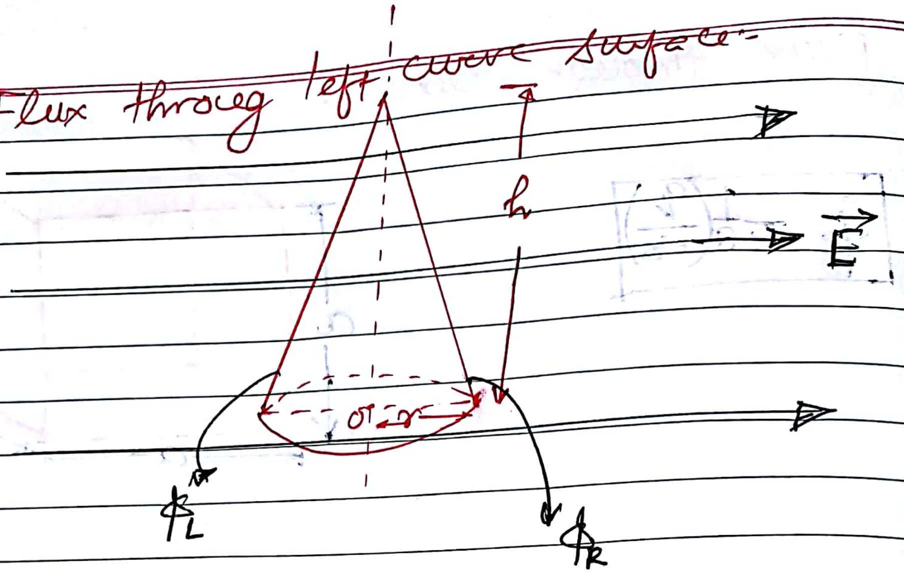
(X) Flux through curve surface:→



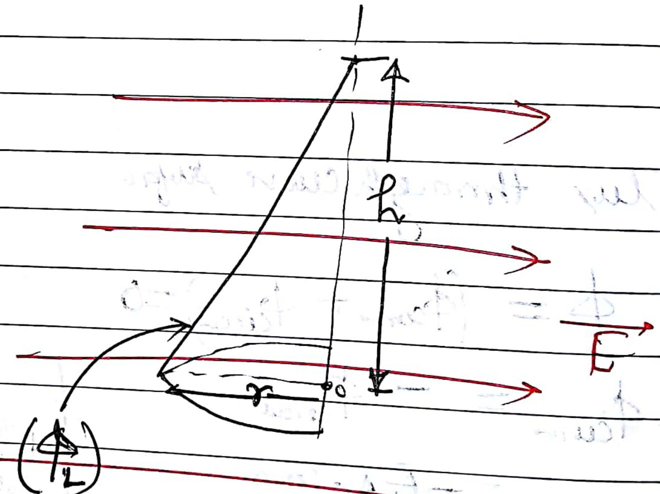
$$\phi = EA\cos 90$$

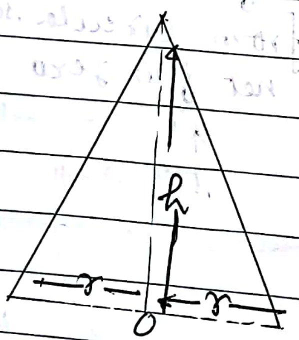
$$\boxed{\phi_{curve} = 0}$$

(XI) ~~Flux throug left curve surface~~



$$\boxed{\begin{aligned}(\phi)_{net} &= 0 \\ (\phi_L + \phi_R) &= 0\end{aligned}}$$





$$\phi_L = -E\left(\tfrac{1}{2} \times 2r \times h\right)$$

$$\boxed{\phi_L = -(Erh)}$$

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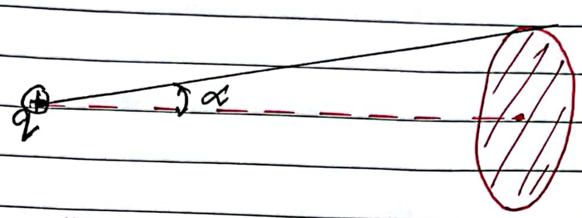
Page

XII

Flux through disc:



$$\boxed{\phi = \frac{q}{2\epsilon_0}\left[1 - \cos\alpha\right]}$$

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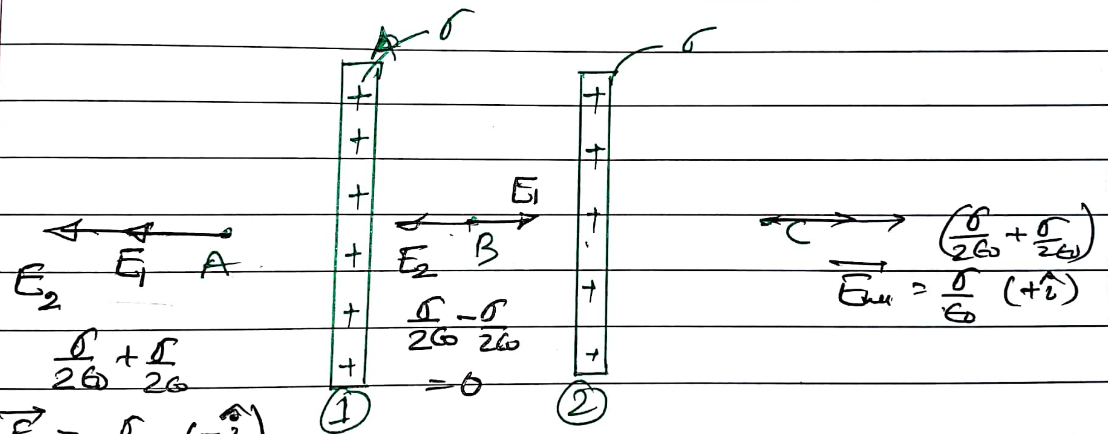
* Q- What is electric field at 'A', 'B' & 'C' point if surface charge distribution is



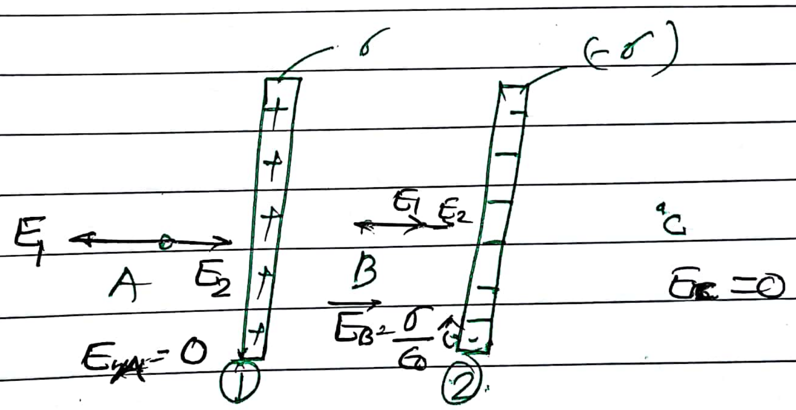
$$\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$

$$\vec{E}_N = \frac{\sigma}{\epsilon_0}(-\hat{i})$$

$$\frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

$$\left(\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}\right)$$

$$\vec{E}_{net} = \frac{\sigma}{\epsilon_0}(+\hat{i})$$



$$E_A = 0$$

$$\vec{E}_B = \frac{\sigma}{\epsilon_0}\hat{i}$$

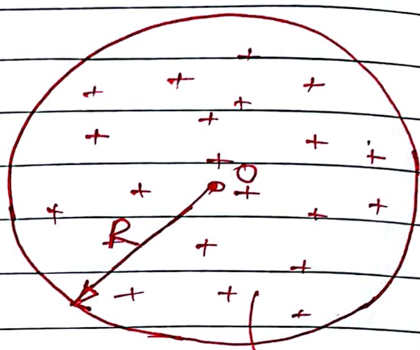
$$E_C = 0$$

Page

Application of Gauss's Law:→

* Electricfield due to uniformly charged distributed volume. [Non-conducting volume]

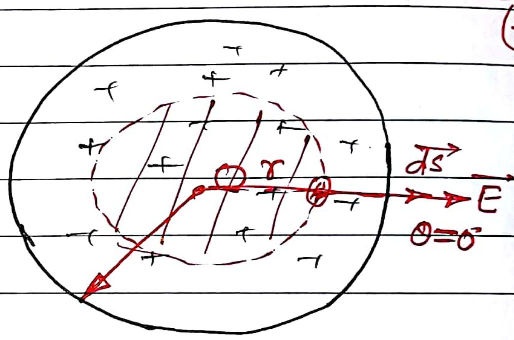
Consider a volume charge density of radius 'R' as shown in figure. If 'ρ' be volumetric charge density.



ρ = Volume charge / Charge density

$= \frac{Q}{\left(\frac{4}{3}\pi R^3\right)}$

Case-I) $(r<R)$



Consider a gaussian surface of radius 'r' as shown in figure. Now, enclosed charge.

$$(q)_{enc} = \left(\rho \times \frac{4}{3}\pi r^3\right)$$

From Gauss's law.

$$\oint \vec{E}\cdot\vec{ds} = \frac{(q)_{enc}}{\epsilon_0}$$

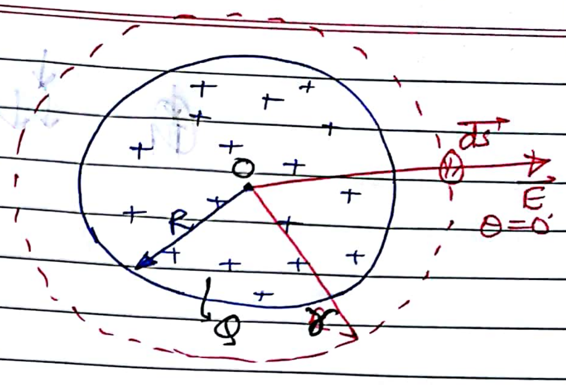
$$E \times \cancel{4\pi} r^{\cancel{2}} = \frac{\rho \times \cancel{\frac{4}{3}\pi} r^{\cancel{3}}}{\epsilon_0}$$

or $\boxed{E_{in} = \left(\frac{\rho}{3\epsilon_0}\right)\cdot r}$

In vector for

$$\boxed{\vec{E}_{in} = \frac{\rho}{3\epsilon_0}\vec{r}}$$

Case-II $\quad r > R$



$$(q)_{enc} = Q$$
$$= \left(\rho \times \frac{4}{3}\pi R^3\right)$$

From Gauss's law,

$$\oint \vec{E}\cdot\vec{ds} = \frac{(q)_{enc}}{\epsilon_0}$$

$$\longrightarrow E \times 4\pi r^2 = \frac{\rho \times \frac{4}{3}\pi R^3}{\epsilon_0}$$

$$\boxed{E = \frac{\rho}{3\epsilon_0}\cdot\frac{R^3}{r^2}}$$

$$\boxed{E_{out} = \frac{Q}{4\pi\epsilon_0 r^2}}$$

Case-III $\quad$ when $r = R$

$$\boxed{E = \frac{Q}{4\pi\epsilon_0 R^2}}$$